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# Wave surface reconstruction based on spline

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**Abstract** The shift method presented recently is a flume wave measurement method based on light refraction and Hilbert transform. A modified shift method is presented based on cubic spline function and least square method. Spline function is applied to both the calculation of the shift and the reconstruction of wave surface. Compared with the original shift method, the modified shift method solves the "zigzag" problem in the calculation of the shift and results in smoother wave surface. The new method can give the wave elevation of arbitrary point.

**Key words** wave surface reconstruction; light refraction; spline; least square method

## 0 Introduction

Wave surface reconstruction, in other words, wave measurement, attaches much importance to wave experiments in laboratory wave flume. Traditionally, the wave gauge of resistance type is used to give the elevation. As an instrusive measurement in the fixed point, it will disturb the wave motion, which restricts the development of the related subjects. So researchers trend to use non-intrusive methods recently.

Slope sensing optical measurement technique is one of the popular fields. Unlike common vision methods[1], it is based on the analysis of the optical distortion created by the fluid surface upon refraction. The advantage of optical measurement is that it can be arranged to avoid the disturbance of the instrument to the wave motion. Since the first method to measure surface slope by light refraction presented by Cox[2], researchers have presented many methods based on laser light[3-5], scanning laser slope gauge[6,7], and the distribution of light intensity[8] or color[9-12] under water. Complicated optical calibration was needed in the above methods, and the result would be sensitive to the noise and nonlinear effects of optical process.

Sun, *et al*[13] developed an easy and low-cost method for wave measurement, which can be called the shift method. The calculation of shift is implemented by Hilbert transform. The shift method is not sensitive to the noise and nonlinear effects in optical process, and can be performed well for measuring wave in the flume. Sun inquired the authors about some mathematic problems in the shift method, and the authors find that spline method can get the same goal, which is presented in the paper.

## 1 Principle of the shift method[13]

Place a plane with black-white stripes on the flume floor as an indication plane. The CCD camera is fixed above the wave flume with its

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optical axis placed vertically. Fig. 1 shows the images of the indication plane under still water and wave condition respectively. The difference of the stripe shapes in the two images is only related to the shape of wave surface, with water body at wave crest acting as a convex and a concave lens at wave trough. Thus the surface slopes can be obtained according to the Snell s Law of Refraction, and then the wave surface can be computed by iterative operations. The detailed algorithm is as follows.

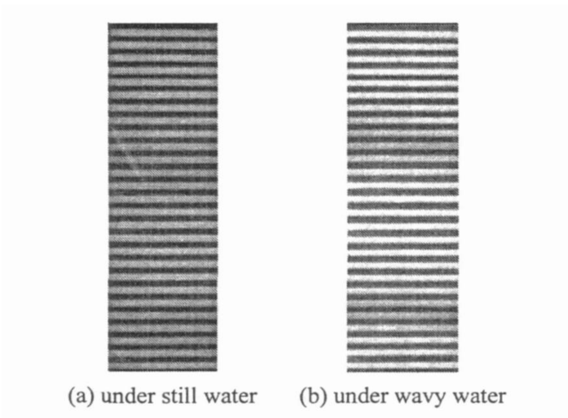



Fig. 1 Experimental images

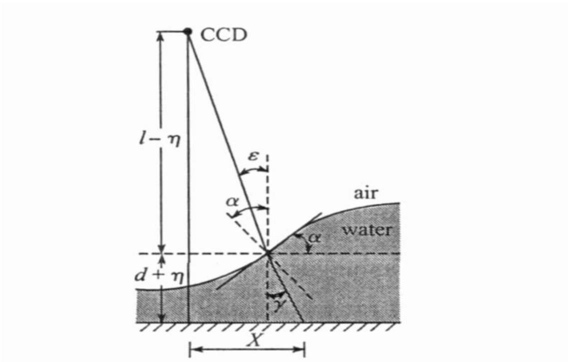



Fig. 2 Optical refraction at the air-water interface[13]

The optical refraction at the air-water interface is shown in Fig. 2. According to the geometrical optics, the refractive index of water $n_w$ can be written as

$$n_w = \frac{\sin(T - X)}{\sin(T - V)} \qquad (1)$$

where T is the tangent angle of wave surface respect to the horizontal, X is the angle of the light entering into CCD camera, and V is the angle between light ray in water and vertical direction at position $X$ on the flume floor, respectively.

Then the slope is given by

$$s = \tan T = \frac{n_w \sin V - \sin X}{n_w \cos V - \cos X} \qquad (2)$$

From the basic geometry in Fig. 2, the shift of $X$ between the wave surface and still water is

$$\Delta X = X - X_0 = (l - Z)\tan X + (d + Z)\tan V - (l\tan X + d\tan V_0) = Z(\tan V - \tan X) + d(\tan V - \tan V_0) \qquad (3)$$

where Z is the wave surface above the still water level, $d$ is the still water depth, $l$ is the distance from the CCD camera to still water level, and $V_0$ is the initial angle under still water condition, respectively. Eq. (3) can be rewritten as

$$\tan V = \frac{Z\tan X + d\tan V_0 + \Delta X}{d + Z} \qquad (4)$$

The wave surface function is obtained by an iterative algorithm.

(1) load the experimental parameters and set initial value;

(2) compute V from Eq. (4);

(3) compute the slope $s$ from Eq. (2);

(4) reconstruct new wave surface, stop the operation if the new wave surface satisfies the precision, or go to (2).

## 2 Application of spline to the reconstruction of wave surface

### 2.1 Application of cubic spline

2.1.1 *Theoretical background* Spline[14, 15] is piecewise polynomial function that can have a locally very simple form, yet at the same time be globally flexible and smooth. Splines are very useful for modeling arbitrary functions, and hence are used widely. Let $S_n(x_1, x_2, \cdots, x_N)$ be the spline space with breakpoint sequence $-\infty = x_0 < x_1 < \cdots < x_N < x_{N+1} = \infty$, then $S(x) \in S_n(x_1, x_2, \cdots, x_N)$ if and only if $\exists\ p_n(x) \in P_n(x)$ and real numbers $c_1, c_2, \cdots, c_N$ satisfy

$$S(x) = p_n(x) + \sum_{j=1}^{N} c_j (x - x_j)_+^n \qquad (5)$$

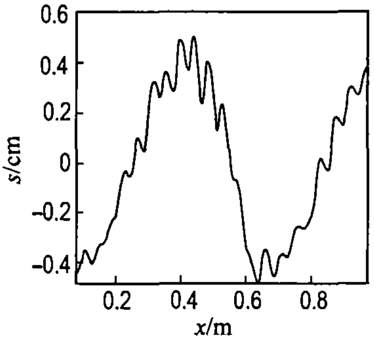
It means that $1, x, x^2, \cdots, x^n, (x - x_1)_+^n, \cdots, (x - x_N)_+^n$ are the base functions of $S_n(x_1, x_2, \cdots, x_N)$.

In practice, cubic spline is used most widely and is also chosen in this paper.

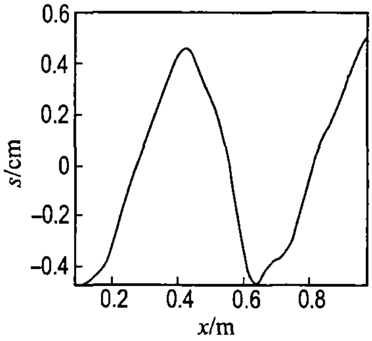
2.1.2 *Calculation of the shift* In Lit. [13], Hilbert transform was applied to compute the shift $\Delta X$ with a "zigzag" effect (Fig. 3(a)[13]). Spline fitting is applied in the present method resulting in a smoother $\Delta X$ function (Fig. 3(b)). Suppose the measurement area is divided into $N$ equal pieces $0 = x_0 < x_1 < \cdots < x_{N-1} < x_N = L$, and $\Delta X$ be a spline function as

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \sum_{j=1}^{N} c_j (x - x_j)_+^n \quad (6)$$

Compare the image under still water with wave condition, the shift in stripe boundary can be got by edge detection[16,17]. An over determined linear system of equations can be obtained by substituting them into Eq. (6). Solving the linear system for the parameters $a_0, \cdots, a_3, c_1, \cdots, c_N$ in the least square sense, the shift $\Delta X$ for arbitrary $x$ is obtained consequently.




(a) shift via Hilbert transform




(b) shift via spline fitting

Fig. 3 The comparison of shift via Hilbert transform and spline fitting

2.1.3 *Reconstruction of the wave surface* Similarly, spline function is applied to the reconstruction of wave surface. The wave surface elevation as a spline function can be supposed as

$$Z(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \sum_{j=1}^{N} c_j (x - x_j)_+^3 \quad (7)$$

Distinctively, here the slopes are known, instead of the elevation of certain point $x$. So the derivative function is obtained as follows:

$$Z'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \sum_{j=1}^{N} 3c_j (x - x_j)_+^2 \quad (8)$$

Substitute the slopes computed by Eq. (2) into Eq. (8), an over determined linear system of equations similar to Section 2.1.2 is also got, then the wave $Z(x)$ is defined only up to the parameter $a_0$ by solving the system.

2.1.4 *Determination of $a_0$* It is quite difficult to determine $a_0$ since only the slopes are considered in the method. Here an estimate method is given: suppose the average wave elevation between wave crest and trough as large scope as possible is equal to 0. This estimate will be invalid if there are no wave crest and trough simultaneously, but fortunately, this problem can be solved by adjusting the position or the focus of the CCD.

## 2.2 Application of B-spline

Another form of spline that can be used to the wave surface reconstruction is B-spline[15], defined by

$$P(t) = \sum_{i=0}^{n} P_i N_{i,k}(t)$$

where $\{P_i\}_0^n$ are the control points, $N_{i,k}(t)$ are basic functions defined recursively as follows:

$$N_{i,1}(x) = \begin{cases} 1; & t_i \leqslant t < t_{i+1} \\ 0; & \text{otherwise} \end{cases}$$

$$N_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t), \quad k > 1$$

where $\{t_i\}_0^{m+k}$ is knot sequence. Similar to Section 2.1, suppose the wave surface elevation is a 4th-degree B-spline function, then the wave surface can also be computed by least square method. The resulting waves are almost the same as Section 2.1. In general, due to its local support property, B-spline may be more accurate than cubic spline.

## 3 Experimental results and analyses

In order to analyze the efficiency of the present method, some experiments with standard functions are conducted. For instance, suppose the wave surface elevation is $y = 0.02\sin(10x)$, compute $\Delta X$ by Eq. (3) and reconstruct the wave surface elevation $Z(x)$ from Eq. (7). The analytical result is almost the same as the standard one. Since it is difficult to obtain accurate $\Delta X$ due to the influence of image noise in practice, Gauss white noise is added to $\Delta X$, the $\Delta X$ and the wave surface are shown in Fig. 4 and Fig. 5 respectively. The analytical wave height is 0.040 8, while the standard one is 0.040 0. Large numbers of testing functions indicate that the spline fitting is efficient.

The present method is applied to experimental images (see Fig. 1) and its results are compared with those of the shift method. The experiments are conducted in a wave flume of 22 m in length, 0.45 m in width and 0.6 m in depth in the State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology. The height of CCD camera to the flume floor is 1.92 m, the width of the stripe is 2 cm and the still water depth is 21 cm. The experimental images were provided by Sun. The shifts and resulting wave surface are shown in Fig. 3 and Fig. 6 respectively. The experimental wave height is 1.47 cm by the present method, which also coincides with the real wave height of 1.5 cm.

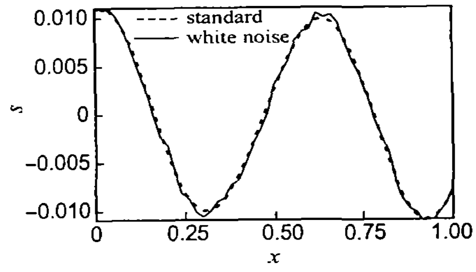



Fig. 4 The comparison of the standard shift and the one with the white noise

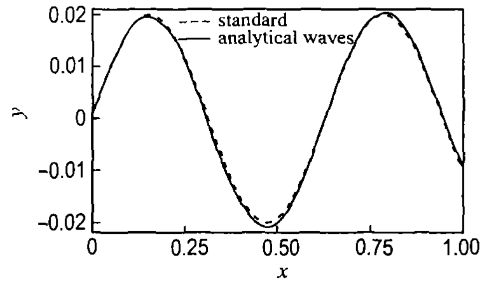



Fig. 5 The comparison of the standard wave surface and the analytical one

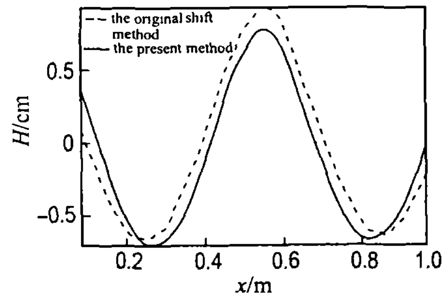



Fig. 6 The comparison of wave surface between shift method and the present method

## 4 Conclusion

In this paper, a novel method for wave surface reconstruction is proposed based on spline fitting and least square methods. The present method can also give the wave surface in the wave flume. Reconstructing function from its slopes is rather unstable, accordingly, highly accurate edge detection (i.e. sub-pixel) is needed in the image processing, so it is believed that a better quality of experimental images will further improve the efficiency.

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provided the experimental images and the results by the shift method.

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# 基于样条的波浪表面重建


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摘要: 最近提出的偏移算法是一种基于光线折射和希尔伯特变换的水槽波浪测量方法. 基于三次样条函数和最小二乘法, 提出了一种改进的偏移算法. 样条函数既用来计算偏移量, 又用做波浪表面重建. 与原偏移算法相比, 改进的偏移算法解决了偏移计算的锯齿问题, 得到了更光滑的波浪表面, 而且新方法可以给出任意一点的波浪高度值.



关键词: 波浪表面重建; 光线折射; 样条; 最小二乘法

中图分类号: O29; TB115 文献标识码: A

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