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基于积分等式的中立系统时滞相关稳定性分析

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摘要： 讨论了具有区间时变时滞及非线性扰动中立系统的稳定性判据问题. 基于Lyapunov-Krasovskii泛函方法和线性矩阵不等式技术，并结合新的积分等式处理交叉乘积项定界问题，得到了新的时滞相关渐近稳定判据. 时滞相关渐近稳定判据用线性矩阵不等式的形式给出，与已有的方法相比，其优点在于更小的保守性. 数值计算表明了结果的有效性和优越性.

关键词： 中立系统；线性矩阵不等式；积分等式；非线性参数扰动

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0 引 言

中立系统是指在状态及状态导数中同时含有时滞的系统，有着较为广泛的应用背景，典型的例子是包含无损传输线的分布网络[1]及人口生态学模型[2]等. 近年来，众多学者对中立型时滞系统的稳定性做了研究[3~5]. 非线性扰动作为不确定的一种表现形式，在时滞系统的稳定性分析过程中备受关注. 常见时域方法有模型变换法[6]、广义模型变换法[7]、自由权矩阵方法[8]、增广 Lyapunov-Krasovskii 泛函[9]等. 一些积分不等式[10~14]常用于处理交叉乘积项的定界. 时滞系统的稳定性判据可分为时滞独立条件[3]和时滞相关条件[4~9、15~19]，由于前者不包含时滞信息，而后者包含时滞信息，一般来说，时滞独立的稳定条件是比较保守的. 此外，因为在网络控制系统中，时滞下界往往要求不能为0[20]，近年来，含有区间时滞的时滞系统的稳定性引起了人们的广泛关注.

本文讨论含有区间时变时滞及非线性扰动的中立系统的稳定性问题. 文献[19]利用Lyapunov-Krasovskii方法结合一些不等式给出该系统的时滞相关稳定性判据，但这些判据仍具有一定的保守性. 例如在处理交叉乘积项的定界时，对于区间时变时滞 $\tau_l \leqslant \tau(t) \leqslant \tau_u$，$\tau_u - \tau(t)$ 和 $\tau(t) - \tau_l$ 均被估计为 $\tau_u - \tau_l$，也就是说 $\tau(t)$ 被估计为 $(\tau_l + \tau_u)/2$，这必将导致其结论的保守性. 事实上，相同保守的估计也存在于文献[8、17]中. 为避免这种保守的估计，本文给出两个新的积分等式；基于 Lyapunov-Krasovskii 泛函方法、线性矩阵不等式技术及两个积分等式，得到保守性更小的时滞相关稳定性判据；最后给出数值例子来说明本文结果的有效性和优越性.

1 问题描述及预备知识

考虑如下中立系统：

$$\begin{cases} \dot{\boldsymbol{x}}(t) - \boldsymbol{C}\dot{\boldsymbol{x}}(t-d(t)) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t-\tau(t)) + \\ \qquad\qquad \boldsymbol{g}(\boldsymbol{x}(t),t) + \\ \qquad\qquad \boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t) + \\ \qquad\qquad \boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t) \\ \boldsymbol{x}(\theta) = \boldsymbol{\varphi}(\theta), \dot{\boldsymbol{x}}(\theta) = \boldsymbol{\phi}(\theta); \theta \in [-\tau, 0] \end{cases} \tag{1}$$

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式中：$\boldsymbol{x}(t)\in\boldsymbol{R}^n$ 为状态向量；$\boldsymbol{\varphi}(\theta)$、$\boldsymbol{\phi}(\theta)$ 为在区间$[-\tau,0]$内连续可微的初始条件，$\tau=\max\{d_{\mathrm{u}},\tau_{\mathrm{u}}\}$；$\boldsymbol{A}$、$\boldsymbol{B}$、$\boldsymbol{C}$ 为已知实常数矩阵；时变中立时滞 $d(t)$ 和时变状态时滞 $\tau(t)$ 满足

$$d(t)\leqslant d_{\mathrm{u}},\dot{d}(t)\leqslant\sigma<1,0\leqslant\tau_{\mathrm{l}}\leqslant\tau(t)\leqslant\tau_{\mathrm{u}},\quad \dot{\tau}(t)\leqslant\mu \tag{2}$$

时变向量函数 $\boldsymbol{g}(\boldsymbol{x}(t),t)\in\boldsymbol{R}^n$，$\boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t)\in\mathbf{R}^n$ 及 $\boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t)\in\mathbf{R}^n$ 未知，但满足

$$\begin{aligned}&\boldsymbol{g}(\mathbf{0},t)=\boldsymbol{h}(\mathbf{0},t)=\boldsymbol{k}(\mathbf{0},t)=\mathbf{0}\\&\|\boldsymbol{g}(\boldsymbol{x}(t),t)\|\leqslant\alpha\|\boldsymbol{x}(t)\|\\&\|\boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t)\|\leqslant\beta\|\boldsymbol{x}(t-\tau(t))\|\\&\|\boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t)\|\leqslant\gamma\|\dot{\boldsymbol{x}}(t-d(t))\|\end{aligned} \tag{3}$$

其中 $\alpha\geqslant0,\beta\geqslant0,\gamma\geqslant0$ 为给定常数. 为了方便分析，记 $\boldsymbol{g}=\boldsymbol{g}(\boldsymbol{x}(t),t)$，$\boldsymbol{h}=\boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t)$，$\boldsymbol{k}=\boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t)$.

首先给出如下积分等式，该等式将在后面的推导中起到重要作用.

引理 1（积分等式）　对于任意适维正定矩阵 $\boldsymbol{Q}>\mathbf{0},\boldsymbol{P}>\mathbf{0}$，适维矩阵 $\boldsymbol{M}$、$\boldsymbol{N}$、$\boldsymbol{X}$，适维向量 $\boldsymbol{\omega}(t)$、$\boldsymbol{\eta}(t)$、$\boldsymbol{x}(t)$ 以及正常数 τ_{u}，有如下积分等式成立：

$$\begin{aligned}(1)\ &-\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}\boldsymbol{x}(s)\mathrm{d}s=\\&2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}(s)\mathrm{d}s+\\&\tau_{\mathrm{u}}\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\omega}(t)-\\&\int_{t-\tau_{\mathrm{u}}}^{t}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\omega}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{P}&\boldsymbol{M}^{\mathrm{T}}\\\boldsymbol{M}&\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\omega}(t)\end{pmatrix}\mathrm{d}s\end{aligned}$$

$$\begin{aligned}(2)\ &-\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s=\\&\tau_{\mathrm{u}}\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{X}\boldsymbol{\eta}(t)+2\boldsymbol{\eta}^{\mathrm{T}}(t)\times\\&\left(\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s+\boldsymbol{N}\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s\right)-\\&\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{Q}&\boldsymbol{M}^{\mathrm{T}}\\\boldsymbol{M}&\boldsymbol{X}\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}\mathrm{d}s-\\&\int_{t-\tau(t)}^{t}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{Q}&\boldsymbol{N}^{\mathrm{T}}\\\boldsymbol{N}&\boldsymbol{X}\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}\mathrm{d}s\end{aligned}$$

证明　(1) $-\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}\boldsymbol{x}(s)\mathrm{d}s=$

$$\begin{aligned}&-\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{P}\boldsymbol{x}(s)\mathrm{d}s-\\&2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}(s)\mathrm{d}s-\\&\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\omega}(t)\mathrm{d}s+\\&2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}(s)\mathrm{d}s+\\&\tau_{\mathrm{u}}\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\omega}(t)=\\&2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}(s)\mathrm{d}s+\\&\tau_{\mathrm{u}}\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{\omega}(t)-\\&\int_{t-\tau_{\mathrm{u}}}^{t}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\omega}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{P}&\boldsymbol{M}^{\mathrm{T}}\\\boldsymbol{M}&\boldsymbol{M}\boldsymbol{P}^{-1}\boldsymbol{M}^{\mathrm{T}}\end{pmatrix}\times\\&\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\omega}(t)\end{pmatrix}\mathrm{d}s\end{aligned}$$

$$\begin{aligned}(2)\ &-\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s=\\&-\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s-\\&2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s-\\&\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{X}\boldsymbol{\eta}(t)\mathrm{d}s-\\&\int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}\boldsymbol{x}(s)\mathrm{d}s-\\&2\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{N}\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s-\\&\int_{t-\tau_{u}}^{t}\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{X}\boldsymbol{\eta}(t)\mathrm{d}s+\\&2\boldsymbol{\eta}^{\mathrm{T}}(t)\left(\boldsymbol{N}\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s+\right.\\&\left.\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s\right)+\int_{t-\tau_{\mathrm{u}}}^{t}\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{X}\boldsymbol{\eta}(t)\mathrm{d}s=\\&-\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{Q}&\boldsymbol{M}^{\mathrm{T}}\\\boldsymbol{M}&\boldsymbol{X}\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}\mathrm{d}s-\\&\int_{t-\tau(t)}^{t}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\boldsymbol{Q}&\boldsymbol{N}^{\mathrm{T}}\\\boldsymbol{N}&\boldsymbol{X}\end{pmatrix}\begin{pmatrix}\boldsymbol{x}(s)\\\boldsymbol{\eta}(t)\end{pmatrix}\mathrm{d}s+\\&2\boldsymbol{\eta}^{\mathrm{T}}(t)\left(\boldsymbol{N}\int_{t-\tau(t)}^{t}\boldsymbol{x}(s)\mathrm{d}s+\right.\\&\left.\boldsymbol{M}\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s\right)+\tau_{\mathrm{u}}\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{X}\boldsymbol{\eta}(t)\end{aligned}$$

一般地，Lyapunov 泛函的导数的上界可以表示为 $\dot{V}(\boldsymbol{x}_t)\leqslant\boldsymbol{\psi}^{\mathrm{T}}(t)\boldsymbol{\Xi}\boldsymbol{\psi}(t)$. 上述引理中的 $\boldsymbol{\xi}(t)$、

$\boldsymbol{\eta}(t)$ 可以取为 $\boldsymbol{\psi}(t)$. 但为了减少计算量,$\boldsymbol{\xi}(t)$、$\boldsymbol{\eta}(t)$ 也可以取 $\boldsymbol{\psi}(t)$ 的一部分.

2 主要结果

利用 Lyapunov-Krasovskii 泛函方法和线性矩阵不等式技术,给出如下时滞相关稳定性判据.

定理 1 给定正常数 α、β、γ、τ_{l}、τ_{u}、μ,则具有时变时滞(2)及非线性扰动(3)的中立系统(1)是渐近稳定的,如果 $\|\boldsymbol{C}\|+\gamma<1$,并存在适维正定或半正定矩阵 $\boldsymbol{X}_1\geqslant\boldsymbol{0},\boldsymbol{X}_2\geqslant\boldsymbol{0},\boldsymbol{W}_i>\boldsymbol{0}(i=1,2,3),\boldsymbol{Z}_j>\boldsymbol{0},\boldsymbol{Q}_j>\boldsymbol{0}(j=1,2,3,4)$,适维矩阵 $\boldsymbol{N}_s(s=1,2,\cdots,7)$,满足如下线性矩阵不等式:

$$\begin{pmatrix}\boldsymbol{\Pi} & \sqrt{\tau_{\mathrm{u}}}\boldsymbol{N}_5 & \sqrt{\tau_{\mathrm{u}}-\tau_{\mathrm{l}}}\boldsymbol{N}_6\\ \sqrt{\tau_{\mathrm{u}}}\boldsymbol{N}_5^{\mathrm{T}} & -\boldsymbol{Z}_2 & \boldsymbol{0}\\ \sqrt{\tau_{\mathrm{u}}-\tau_{\mathrm{l}}}\boldsymbol{N}_6^{\mathrm{T}} & \boldsymbol{0} & -\boldsymbol{Z}_4\end{pmatrix}<\boldsymbol{0}\quad(4)$$

式中:$\boldsymbol{\Pi}=\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_2+\boldsymbol{\Phi}_2^{\mathrm{T}}+\boldsymbol{\Phi}_3+\boldsymbol{\Phi}_3^{\mathrm{T}}+\boldsymbol{\Phi}_4$,其中 $\boldsymbol{\Phi}_1=\mathrm{diag}\{\boldsymbol{\Lambda}_1,\boldsymbol{\Lambda}_2,\boldsymbol{Q}_1-\boldsymbol{Q}_2,-\boldsymbol{Q}_1,\boldsymbol{0},\boldsymbol{0},\gamma^2\boldsymbol{W}_3-(1-\sigma)\boldsymbol{Q}_3,\boldsymbol{Z},-\boldsymbol{W}_1,-\boldsymbol{W}_2,-\boldsymbol{W}_3\}$,$\boldsymbol{\Lambda}_1=(\tau_{\mathrm{u}}-\tau_{\mathrm{l}})(\boldsymbol{Z}_3+\boldsymbol{Z}_4)+\boldsymbol{Q}_2+\boldsymbol{Q}_4+\alpha^2\boldsymbol{W}_1$, $\boldsymbol{\Lambda}_2=\beta^2\boldsymbol{W}_2-(1-\mu)\boldsymbol{Q}_4$,$\boldsymbol{Z}=(\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{Z}_1+\tau_{\mathrm{u}}\boldsymbol{Z}_2+\boldsymbol{Q}_3$; $\boldsymbol{\Phi}_2=(\boldsymbol{N}_5\quad\boldsymbol{E}_3^{\mathrm{T}}(\boldsymbol{N}_1-\boldsymbol{N}_2)\quad\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{N}_2\quad-\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{N}_1-\boldsymbol{N}_5\quad\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{N}_4+\boldsymbol{N}_6\quad\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{N}_3+\boldsymbol{N}_6\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{0})$,

$$\boldsymbol{E}_3=\begin{pmatrix}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{pmatrix},$$

$$\boldsymbol{E}_4=\begin{pmatrix}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{pmatrix};$$

$\boldsymbol{\Phi}_3=\boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}_2+\boldsymbol{N}_7\boldsymbol{\mathscr{A}}$,

$$\boldsymbol{E}_1=\begin{pmatrix}\boldsymbol{A}&\boldsymbol{B}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{C}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{I}&\boldsymbol{I}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&-\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{pmatrix},$$

$$\boldsymbol{E}_2=\begin{pmatrix}\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\\ \boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{I}&\boldsymbol{I}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}&\boldsymbol{0}\end{pmatrix},$$

$\boldsymbol{\mathscr{A}}=(\boldsymbol{A}\quad\boldsymbol{B}\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{0}\quad\boldsymbol{C}\quad-\boldsymbol{I}\quad\boldsymbol{I}\quad\boldsymbol{I}\quad\boldsymbol{I})$;

$\boldsymbol{\Phi}_4=(\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{X}_1\boldsymbol{E}_3+(\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{X}_2\boldsymbol{E}_4$.

$$\boldsymbol{\Omega}_1=\begin{pmatrix}\boldsymbol{Z}_1&\boldsymbol{N}_1^{\mathrm{T}}\\ \boldsymbol{N}_1&\boldsymbol{X}_1\end{pmatrix}\geqslant\boldsymbol{0},\boldsymbol{\Omega}_2=\begin{pmatrix}\boldsymbol{Z}_1&\boldsymbol{N}_2^{\mathrm{T}}\\ \boldsymbol{N}_2&\boldsymbol{X}_1\end{pmatrix}\geqslant\boldsymbol{0},$$

$$\boldsymbol{\Omega}_3=\begin{pmatrix}\boldsymbol{Z}_3&\boldsymbol{N}_3^{\mathrm{T}}\\ \boldsymbol{N}_3&\boldsymbol{X}_2\end{pmatrix}\geqslant\boldsymbol{0},\boldsymbol{\Omega}_4=\begin{pmatrix}\boldsymbol{Z}_3&\boldsymbol{N}_4^{\mathrm{T}}\\ \boldsymbol{N}_4&\boldsymbol{X}_2\end{pmatrix}\geqslant\boldsymbol{0}\quad(5)$$

证明 选择如下 Lyapunov-Krasovskii 泛函

$$V(\boldsymbol{x}_t)\triangleq V_1(\boldsymbol{x}_t)+V_2(\boldsymbol{x}_t)+V_3(\boldsymbol{x}_t)+V_4(\boldsymbol{x}_t)\quad(6)$$

其中

$$V_1(\boldsymbol{x}_t)=\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\eta}(t)$$

$$\boldsymbol{\eta}^{\mathrm{T}}(t)=\left(\boldsymbol{x}^{\mathrm{T}}(t)\quad\left(\int_{t-\tau_{\mathrm{u}}}^{t-\tau_{\mathrm{l}}}\boldsymbol{x}(s)\mathrm{d}s\right)^{\mathrm{T}}\right)$$

$$\begin{aligned}V_2(\boldsymbol{x}_t)=&\int_{t-\tau_{\mathrm{u}}}^{t-\tau_{\mathrm{l}}}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s)\mathrm{d}s+\\&\int_{t-\tau_{\mathrm{l}}}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_2\boldsymbol{x}(s)\mathrm{d}s+\\&\int_{t-d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_3\dot{\boldsymbol{x}}(s)\mathrm{d}s+\\&\int_{t-\tau(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_4\boldsymbol{x}(s)\mathrm{d}s\end{aligned}$$

$$\begin{aligned}V_3(\boldsymbol{x}_t)=&\int_{-\tau_{\mathrm{u}}}^{-\tau_{\mathrm{l}}}\int_{t+s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}t+\\&\int_{-\tau_{\mathrm{u}}}^{0}\int_{t+s}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}t\end{aligned}$$

$$V_4(\boldsymbol{x}_t)=\int_{-\tau_{\mathrm{u}}}^{-\tau_{\mathrm{l}}}\int_{t+s}^{t}\boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{Z}_3+\boldsymbol{Z}_4)\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}t$$

计算 $V_i(\boldsymbol{x}_t),i=1,2,3,4$ 对时间 t 的导数分别为

$$\begin{aligned}\dot{V}_1(\boldsymbol{x}_t)=&2\begin{pmatrix}\dot{\boldsymbol{x}}(t)\\ \boldsymbol{x}(t-\tau_{\mathrm{l}})-\boldsymbol{x}(t-\tau_{\mathrm{u}})\end{pmatrix}^{\mathrm{T}}\times\\&\boldsymbol{P}\begin{pmatrix}\boldsymbol{x}(t)\\ \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)}\boldsymbol{x}(s)\mathrm{d}s+\int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}}\boldsymbol{x}(s)\mathrm{d}s\end{pmatrix}=\\&\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{E}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}_2+\boldsymbol{E}_2^{\mathrm{T}}\boldsymbol{P}\boldsymbol{E}_1)\boldsymbol{\xi}(t)\end{aligned}\quad(7)$$

$$\begin{aligned}\dot{V}_2(\boldsymbol{x}_t)\leqslant&\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{Q}_2+\boldsymbol{Q}_4)\boldsymbol{x}(t)+\\&\boldsymbol{x}^{\mathrm{T}}(t-\tau_{\mathrm{l}})(\boldsymbol{Q}_1-\boldsymbol{Q}_2)\boldsymbol{x}(t-\tau_{\mathrm{l}})-\\&\boldsymbol{x}^{\mathrm{T}}(t-\tau_{\mathrm{u}})\boldsymbol{Q}_1\boldsymbol{x}(t-\tau_{\mathrm{u}})-\\&(1-\sigma)\dot{\boldsymbol{x}}^{\mathrm{T}}(t-d(t))\boldsymbol{Q}_3\dot{\boldsymbol{x}}(t-\\&d(t))+\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{Q}_3\dot{\boldsymbol{x}}(t)-(1-\mu)\times\\&\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{Q}_4\boldsymbol{x}(t-\tau(t))\end{aligned}\quad(8)$$

由引理 1 可得

$$\dot{V}_3(\boldsymbol{x}_t) = \dot{\boldsymbol{x}}^{\mathrm{T}}(t)((\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{Z}_1+\tau_{\mathrm{u}}\boldsymbol{Z}_2)\dot{\boldsymbol{x}}(t) - \int_{t-\tau_{\mathrm{u}}}^{t-\tau_{\mathrm{l}}} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s - \int_{t-\tau_{\mathrm{u}}}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s = \dot{\boldsymbol{x}}^{\mathrm{T}}(t)((\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{Z}_1+\tau_{\mathrm{u}}\boldsymbol{Z}_2)\dot{\boldsymbol{x}}(t) + \boldsymbol{\xi}^{\mathrm{T}}(t)((\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{X}_1\boldsymbol{E}_3 + \tau_{\mathrm{u}}\boldsymbol{N}_5\boldsymbol{Z}_2^{-1}\boldsymbol{N}_5^{\mathrm{T}})\boldsymbol{\xi}(t) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{N}_1 \times (\boldsymbol{x}(t-\tau(t)) - \boldsymbol{x}(t-\tau_{\mathrm{u}})) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{E}_3^{\mathrm{T}}\boldsymbol{N}_2(\boldsymbol{x}(t-\tau_{\mathrm{l}}) - \boldsymbol{x}(t-\tau(t))) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}_5(\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau_{\mathrm{u}})) - \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{\rho}_1^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_1\boldsymbol{\rho}_1(t,s)\mathrm{d}s - \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{\rho}_1^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_2\boldsymbol{\rho}_1(t,s)\mathrm{d}s - \int_{t-\tau_{\mathrm{u}}}^{t} \boldsymbol{\rho}_2^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_5\boldsymbol{\rho}_2(t,s)\mathrm{d}s \tag{9}$$

$$\dot{V}_4(\boldsymbol{x}_t) = (\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{Z}_3+\boldsymbol{Z}_4)\boldsymbol{x}(t) - \int_{t-\tau_{\mathrm{u}}}^{t-\tau_{\mathrm{l}}} \boldsymbol{x}^{\mathrm{T}}(s)(\boldsymbol{Z}_3+\boldsymbol{Z}_4)\boldsymbol{x}(s)\mathrm{d}s = (\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{x}^{\mathrm{T}}(t)(\boldsymbol{Z}_3+\boldsymbol{Z}_4)\boldsymbol{x}(t) + (\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{X}_2\boldsymbol{E}_4 + \boldsymbol{N}_6\boldsymbol{Z}_4^{-1}\boldsymbol{N}_6^{\mathrm{T}})\boldsymbol{\xi}(t) + 2\boldsymbol{\xi}^{\mathrm{T}}(t)((\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{N}_4 + \boldsymbol{N}_6)\int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{x}(s)\mathrm{d}s + (\boldsymbol{E}_4^{\mathrm{T}}\boldsymbol{N}_3 + \boldsymbol{N}_6)\int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{x}(s)\mathrm{d}s) - \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{\rho}_3^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_3\boldsymbol{\rho}_3(t,s)\mathrm{d}s - \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{\rho}_3^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_4\boldsymbol{\rho}_3(t,s)\mathrm{d}s - \int_{t-\tau_{\mathrm{u}}}^{t-\tau_{\mathrm{l}}} \boldsymbol{\rho}_4^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_6\boldsymbol{\rho}_4(t,s)\mathrm{d}s \tag{10}$$

式中：$\boldsymbol{\Omega}_i(i=1,2,3,4)$，在式(5)中已给出；

$$\boldsymbol{\Omega}_5 = \begin{pmatrix} \boldsymbol{Z}_2 & \boldsymbol{N}_5^{\mathrm{T}} \\ \boldsymbol{N}_5 & \boldsymbol{N}_5\boldsymbol{Z}_2^{-1}\boldsymbol{N}_5^{\mathrm{T}} \end{pmatrix}$$

$$\boldsymbol{\Omega}_6 = \begin{pmatrix} \boldsymbol{Z}_4 & \boldsymbol{N}_6^{\mathrm{T}} \\ \boldsymbol{N}_6 & \boldsymbol{N}_6\boldsymbol{Z}_4^{-1}\boldsymbol{N}_6^{\mathrm{T}} \end{pmatrix}$$

$$\boldsymbol{\rho}_1(t,s) = \begin{pmatrix} \dot{\boldsymbol{x}}(s) \\ \boldsymbol{\eta}_1(t) \end{pmatrix}$$

其中 $\boldsymbol{\eta}_1^{\mathrm{T}}(t) = (\boldsymbol{x}^{\mathrm{T}}(t) \quad \boldsymbol{x}^{\mathrm{T}}(t-\tau(t)) \quad \boldsymbol{x}^{\mathrm{T}}(t-\tau_{\mathrm{l}}) \quad \boldsymbol{x}^{\mathrm{T}}(t-\tau_{\mathrm{u}}))$；

$$\boldsymbol{\rho}_2(t,s) = \begin{pmatrix} \dot{\boldsymbol{x}}(s) \\ \boldsymbol{\xi}(t) \end{pmatrix}$$

其中 $\boldsymbol{\xi}^{\mathrm{T}}(t) = \Big(\boldsymbol{\eta}_1^{\mathrm{T}}(t) \quad \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s \quad \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s \quad \dot{\boldsymbol{x}}^{\mathrm{T}}(t-d(t)) \quad \dot{\boldsymbol{x}}^{\mathrm{T}}(t) \quad \boldsymbol{g}^{\mathrm{T}} \quad \boldsymbol{h}^{\mathrm{T}} \quad \boldsymbol{k}^{\mathrm{T}}\Big)$；

$$\boldsymbol{\rho}_3(t,s) = \begin{pmatrix} \boldsymbol{x}(s) \\ \boldsymbol{\eta}_2(t) \end{pmatrix}$$

其中 $\boldsymbol{\eta}_2^{\mathrm{T}}(t) = \Big(\boldsymbol{x}^{\mathrm{T}}(t) \quad \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s \quad \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\Big)$；

$$\boldsymbol{\rho}_4(t,s) = \begin{pmatrix} \boldsymbol{x}(s) \\ \boldsymbol{\xi}(t) \end{pmatrix}$$

另一方面，由式(3)，有

$$\boldsymbol{g}^{\mathrm{T}}\boldsymbol{g} \leqslant \alpha^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t)$$
$$\boldsymbol{h}^{\mathrm{T}}\boldsymbol{h} \leqslant \beta^2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{x}(t-\tau(t))$$
$$\boldsymbol{k}^{\mathrm{T}}\boldsymbol{k} \leqslant \gamma^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t-d(t))\dot{\boldsymbol{x}}(t-d(t))$$

故存在适维正定矩阵 $\boldsymbol{W}_i(i=1,2,3)$，使得如下不等式成立：

$$b_1(t) = \alpha^2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{W}_1\boldsymbol{x}(t) - \boldsymbol{g}^{\mathrm{T}}\boldsymbol{W}_1\boldsymbol{g} \geqslant 0,$$
$$b_2(t) = \beta^2\boldsymbol{x}^{\mathrm{T}}(t-\tau(t))\boldsymbol{W}_2\boldsymbol{x}(t-\tau(t)) - \boldsymbol{h}^{\mathrm{T}}\boldsymbol{W}_2\boldsymbol{h} \geqslant 0,$$
$$b_3(t) = \gamma^2\dot{\boldsymbol{x}}^{\mathrm{T}}(t-d(t))\boldsymbol{W}_3\dot{\boldsymbol{x}}(t-d(t)) - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{W}_3\boldsymbol{k} \geqslant 0 \tag{11}$$

类似于文献[4]，对于任意适维的矩阵 $\boldsymbol{N}_7$ 有

$$b_4(t) = 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}_7(\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t-\tau(t)) - \dot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t-d(t)) + \boldsymbol{g} + \boldsymbol{h} + \boldsymbol{k}) = 0 \tag{12}$$

因此，由式(7)～(12)可得

$$\dot{V}(\boldsymbol{x}_t) \leqslant \dot{V}_1(\boldsymbol{x}_t) + \dot{V}_2(\boldsymbol{x}_t) + \dot{V}_3(\boldsymbol{x}_t) + \dot{V}_4(\boldsymbol{x}_t) + \sum_{i=1}^{4} b_i(t) \leqslant \boldsymbol{\xi}^{\mathrm{T}}(t)[\boldsymbol{\Pi} + \tau_{\mathrm{u}}\boldsymbol{N}_5\boldsymbol{Z}_2^{-1}\boldsymbol{N}_5^{\mathrm{T}} + (\tau_{\mathrm{u}}-\tau_{\mathrm{l}})\boldsymbol{N}_6\boldsymbol{Z}_4^{-1}\boldsymbol{N}_6^{\mathrm{T}}]\boldsymbol{\xi}(t) - \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{\rho}_1^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_1\boldsymbol{\rho}_1(t,s)\mathrm{d}s - \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{\rho}_1^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_2\boldsymbol{\rho}_1(t,s)\mathrm{d}s - \int_{t-\tau_{\mathrm{u}}}^{t-\tau(t)} \boldsymbol{\rho}_3^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_3\boldsymbol{\rho}_3(t,s)\mathrm{d}s - \int_{t-\tau(t)}^{t-\tau_{\mathrm{l}}} \boldsymbol{\rho}_3^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_4\boldsymbol{\rho}_3(t,s)\mathrm{d}s -$$

$$\int_{t-\tau_u}^{t} \boldsymbol{\rho}_2^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_5\boldsymbol{\rho}_2(t,s)\mathrm{d}s - \int_{t-\tau_u}^{t-\tau_l} \boldsymbol{\rho}_4^{\mathrm{T}}(t,s)\boldsymbol{\Omega}_6\boldsymbol{\rho}_4(t,s)\mathrm{d}s \tag{13}$$

注意到,如果 $\boldsymbol{Z}_2 > \boldsymbol{0}, \boldsymbol{Z}_4 > \boldsymbol{0}, \boldsymbol{\Omega}_i \geqslant \boldsymbol{0}(i=1,2,3,4)$ 成立,式(13)中的最后 6 项都小于或等于 0. 又由 Schur 补公式知,$\boldsymbol{\Pi} + \tau_u \boldsymbol{N}_5 \boldsymbol{Z}_2^{-1} \boldsymbol{N}_5^{\mathrm{T}} + (\tau_u - \tau_l)\boldsymbol{N}_6 \boldsymbol{Z}_4^{-1} \boldsymbol{N}_6^{\mathrm{T}} < \boldsymbol{0}$ 等价于式(4)成立. 故对任意 $\lambda > 0$ 有 $\dot{V}(\boldsymbol{x}_t) \leqslant -\lambda \| \boldsymbol{x}(t) \|^2$ 成立. 故中立系统(1)渐近稳定. □

由定理的证明过程可见:(1)未使用任何模型变换;(2)利用新的积分等式去处理交叉乘积项的定界问题,不额外引入保守性. 因此,上述结论优于现有文献结论.

以下考虑当时滞下界 $\tau_l = 0$ 时,具有非线性扰动(3)的系统(1)的稳定性.

推论 1 给定正常数 α、β、γ、τ_u、μ,则时变时滞满足 $0 \leqslant \tau(t) \leqslant \tau_u$,非线性扰动满足式(3)的中立系统(1)是渐近稳定的,如果 $\| \boldsymbol{C} \| + \gamma < 1$ 并且存在适维的正定或半正定矩阵 $\boldsymbol{X}_3 \geqslant \boldsymbol{0}, \boldsymbol{X}_4 \geqslant \boldsymbol{0}$, $\boldsymbol{Z}_i > \boldsymbol{0}, \boldsymbol{Q}_i > \boldsymbol{0}, \boldsymbol{W}_i > \boldsymbol{0}(i=1,2,3)$ 及适维矩阵 $\boldsymbol{S}_j(j=1,2,\cdots,7)$,使得如下线性矩阵不等式成立:

$$\begin{pmatrix} \overline{\boldsymbol{\Phi}}_1 + \overline{\boldsymbol{\Phi}}_2 + \overline{\boldsymbol{\Phi}}_2^{\mathrm{T}} + \overline{\boldsymbol{\Phi}}_3 + \overline{\boldsymbol{\Phi}}_3^{\mathrm{T}} + \overline{\boldsymbol{\Phi}}_4 & \sqrt{\tau_u}\boldsymbol{S}_5 & \sqrt{\tau_u}\boldsymbol{S}_6 \\ \sqrt{\tau_u}\boldsymbol{S}_5^{\mathrm{T}} & -\boldsymbol{Z}_2 & \boldsymbol{0} \\ \sqrt{\tau_u}\boldsymbol{S}_6^{\mathrm{T}} & \boldsymbol{0} & -\boldsymbol{Z}_4 \end{pmatrix} < 0 \tag{14}$$

$$\overline{\boldsymbol{\Omega}}_1 = \begin{pmatrix} \boldsymbol{Z}_1 & \boldsymbol{S}_1^{\mathrm{T}} \\ \boldsymbol{S}_1 & \boldsymbol{X}_3 \end{pmatrix} \geqslant \boldsymbol{0},\ \overline{\boldsymbol{\Omega}}_2 = \begin{pmatrix} \boldsymbol{Z}_1 & \boldsymbol{S}_2^{\mathrm{T}} \\ \boldsymbol{S}_2 & \boldsymbol{X}_3 \end{pmatrix} \geqslant \boldsymbol{0},$$

$$\overline{\boldsymbol{\Omega}}_3 = \begin{pmatrix} \boldsymbol{Z}_3 & \boldsymbol{S}_3^{\mathrm{T}} \\ \boldsymbol{S}_3 & \boldsymbol{X}_4 \end{pmatrix} \geqslant \boldsymbol{0},\ \overline{\boldsymbol{\Omega}}_4 = \begin{pmatrix} \boldsymbol{Z}_3 & \boldsymbol{S}_4^{\mathrm{T}} \\ \boldsymbol{S}_4 & \boldsymbol{X}_4 \end{pmatrix} \geqslant \boldsymbol{0} \tag{15}$$

式中

$$\overline{\boldsymbol{\Phi}}_1 = \mathrm{diag}\{\overline{\boldsymbol{\Lambda}}_1, \overline{\boldsymbol{\Lambda}}_2, -\boldsymbol{Q}_1, \boldsymbol{0}, \boldsymbol{0}, \gamma^2\boldsymbol{W}_3 - (1-\sigma)\boldsymbol{Q}_3, \boldsymbol{Z}, -\boldsymbol{W}_1, -\boldsymbol{W}_2, -\boldsymbol{W}_3\};$$

$$\overline{\boldsymbol{\Phi}}_2 = (\overline{\boldsymbol{E}}_3^{\mathrm{T}}\boldsymbol{S}_2 + \boldsymbol{S}_5 \quad \overline{\boldsymbol{E}}_3^{\mathrm{T}}(\boldsymbol{S}_1 - \boldsymbol{S}_2) \quad -\overline{\boldsymbol{E}}_3^{\mathrm{T}}\boldsymbol{S}_1 - \boldsymbol{S}_5 \quad \overline{\boldsymbol{E}}_4^{\mathrm{T}}\boldsymbol{S}_4 + \boldsymbol{S}_6 \quad \overline{\boldsymbol{E}}_4^{\mathrm{T}}\boldsymbol{S}_3 + \boldsymbol{S}_6 \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0})$$

其中 $\overline{\boldsymbol{E}}_3 = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}$,

$$\overline{\boldsymbol{E}}_4 = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}$$

$$\overline{\boldsymbol{\Phi}}_3 = \overline{\boldsymbol{E}}_1^{\mathrm{T}}\boldsymbol{P}\overline{\boldsymbol{E}}_2 + \boldsymbol{S}_7\overline{\boldsymbol{\mathscr{A}}}$$

其中 $\overline{\boldsymbol{E}}_1 = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{C} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}$,

$$\overline{\boldsymbol{E}}_2 = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{pmatrix}$$

$$\overline{\boldsymbol{\mathscr{A}}} = (\boldsymbol{A} \quad \boldsymbol{B} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{0} \quad \boldsymbol{C} \quad -\boldsymbol{I} \quad \boldsymbol{I} \quad \boldsymbol{I} \quad \boldsymbol{I});$$

$$\overline{\boldsymbol{\Phi}}_4 = \tau_u\overline{\boldsymbol{E}}_3^{\mathrm{T}}\boldsymbol{X}_3\overline{\boldsymbol{E}}_3 + \tau_u\overline{\boldsymbol{E}}_4^{\mathrm{T}}\boldsymbol{X}_4\overline{\boldsymbol{E}}_4;$$

$$\overline{\boldsymbol{Z}} = \tau_u(\boldsymbol{Z}_1 + \boldsymbol{Z}_2) + \boldsymbol{Q}_3;$$

$$\overline{\boldsymbol{\Lambda}}_1 = \tau_u(\boldsymbol{Z}_3 + \boldsymbol{Z}_4) + \boldsymbol{Q}_1 + \boldsymbol{Q}_2 + \alpha^2\boldsymbol{W}_1;$$

$$\overline{\boldsymbol{\Lambda}}_2 = \beta^2\boldsymbol{W}_2 + (\mu - 1)\boldsymbol{Q}_2$$

证明 选择如下 Lyapunov-Krasovskii 泛函

$$\overline{V}(\boldsymbol{x}_t) \triangleq \overline{V}_1(\boldsymbol{x}_t) + \overline{V}_2(\boldsymbol{x}_t) + \overline{V}_3(\boldsymbol{x}_t)$$

其中

$$\overline{V}_1(\boldsymbol{x}_t) = \begin{pmatrix} \boldsymbol{x}(t) \\ \int_{t-\tau_u}^{t} \boldsymbol{x}(s)\mathrm{d}s \end{pmatrix}^{\mathrm{T}} \boldsymbol{P} \begin{pmatrix} \boldsymbol{x}(t) \\ \int_{t-\tau_u}^{t} \boldsymbol{x}(s)\mathrm{d}s \end{pmatrix}$$

$$\overline{V}_2(\boldsymbol{x}_t) = \int_{t-\tau_u}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_1\boldsymbol{x}(s)\mathrm{d}s + \int_{t-d(t)}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Q}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s + \int_{t-\tau(t)}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_3\boldsymbol{x}(s)\mathrm{d}s$$

$$\overline{V}_3(\boldsymbol{x}_t) = \int_{-\tau_u}^{0}\int_{t+s}^{t} \dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{Z}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}t + \int_{-\tau_u}^{0}\int_{t+s}^{t} \boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Z}_3\boldsymbol{x}(s)\mathrm{d}s\mathrm{d}t$$

采用与定理 1 类似的证明方法,即得. □

令 $\boldsymbol{C} = \boldsymbol{0}, \boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)), t) = \boldsymbol{0}$,时变时滞满足 $0 \leqslant \tau(t) \leqslant \tau_u$ 的中立系统(1)变为

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{x}(t-\tau(t)) + \boldsymbol{g}(\boldsymbol{x}(t), t) + \boldsymbol{h}(\boldsymbol{x}(t-\tau(t)), t) \\ \boldsymbol{x}(t) = \boldsymbol{\varphi}(t); t \in [-\tau_u, 0] \end{cases} \tag{16}$$

文献[6、7、15～18],相继给出系统(16)的稳定判据,但这些结论具有一定的保守性. 上述推论令 $\boldsymbol{C}=\boldsymbol{0}$ 即可得到保守性更小的系统(16)的稳定判据.

3 数值例子

本章给出几个数值例子,来说明本文结论的

有效性与优越性.

例 1　考虑时变时滞满足 $0\leqslant\tau(t)\leqslant\tau_u$ 中立系统(1),其中

$$\boldsymbol{A}=\begin{pmatrix}-1.2 & 0.1\\ -0.1 & -1\end{pmatrix},\ \boldsymbol{B}=\begin{pmatrix}-0.6 & 0.7\\ -1.0 & -0.8\end{pmatrix},$$

$$\boldsymbol{C}=\begin{pmatrix}c & 0\\ 0 & c\end{pmatrix}$$

且

$$\|\boldsymbol{g}(\boldsymbol{x}(t),t)\|\leqslant\alpha\|\boldsymbol{x}(t)\|$$

$$\|\boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t)\|\leqslant\beta\|\boldsymbol{x}(t-\tau(t))\|$$

$$\|\boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t)\|\leqslant\gamma\|\dot{\boldsymbol{x}}(t-d(t))\|;$$

$$\alpha\geqslant0,\beta\geqslant0,\gamma\geqslant0$$

表 1、2 和 3 分别给出了不同情况下计算时滞上限的比较结果.

表 1　不同方法计算的时滞上界 ($\sigma=0$, $\beta=0.1$, $\mu=0.5$, $c=0.1$)

Tab. 1　The achieved delay upper bounds by different methods ($\sigma=0$, $\beta=0.1$, $\mu=0.5$, $c=0.1$)

γ	$\tau_u(\alpha=0)$					$\tau_u(\alpha=0.1)$				
	文献[15]	文献[17]	文献[19]	文献[8]	本文推论	文献[15]	文献[17]	文献[19]	文献[8]	本文推论
0	0.932 8	0.948 8	0.948 3	0.983 9	1.214 5	0.814 8	0.840 8	0.840 8	0.875 2	1.079 9
0.1	0.740 2	0.769 5	0.811 9	0.802 4	0.999 9	0.643 9	0.684 1	0.722 2	0.716 6	0.901 7
0.2	0.563 7	0.608 7	0.697 5	0.639 2	0.808 7	0.486 4	0.542 0	0.621 4	0.572 7	0.731 3
0.3	0.404 2	0.466 7	0.600 9	0.494 1	0.636 1	0.343 3	0.414 4	0.535 9	0.443 8	0.573 2

表 2　不同方法计算的时滞上界 ($\sigma=0$, $\beta=0.1$, $c=0.1$, μ 任意)

Tab. 2　The achieved delay upper bounds by different methods ($\sigma=0$, $\beta=0.1$, $c=0.1$, any μ)

γ	$\tau_u(\alpha=0)$			$\tau_u(\alpha=0.1)$		
	文献[19]	文献[18]	本文推论	文献[19]	文献[18]	本文推论
0	0.651 0	0.812 2	1.113 0	0.630 2	0.771 1	1.050 8
0.1	0.609 1	0.697 2	0.955 8	0.584 6	0.657 7	0.889 7
0.2	0.561 2	0.583 8	0.795 0	0.537 1	0.546 2	0.729 6
0.3	0.495 4	0.472 2	0.629 5	0.475 9	0.436 8	0.572 9

表 3　不同方法计算的时滞上界 ($c=0$, $\beta=0.1$)

Tab. 3　The achieved delay upper bounds by different methods ($c=0$, $\beta=0.1$)

α	μ	τ_u							
		文献[6]	文献[7]	文献[15]	文献[16]	文献[17]	文献[18]	文献[8]	本文推论
0	0.5	0.546 7	0.674 3	1.136 5	1.142 4	1.142 0	1.142 5	1.184 9	1.397 9
0	任意	—	—	—	—	—	0.735 5	0.928 4	1.282 0
0.1	0.5	0.495 0	0.571 5	0.995 2	1.009 7	1.009 0	1.009 7	1.052 1	1.248 8
0.1	任意	—	—	—	—	—	0.714 7	0.886 5	1.213 3

由表 1 和 2 可以看出,随着 γ 的增大,时滞上界 τ_u 减小. 而且,本文方法计算时滞上界大于相关文献方法,说明本文结果保守性更小.

例 2　考虑系统(1),其中

$$\boldsymbol{A}=\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix},\ \boldsymbol{B}=\begin{pmatrix}0 & 0.4\\ 0.4 & 0\end{pmatrix},$$

$$\boldsymbol{C}=\begin{pmatrix}0.1 & 0\\ 0 & 0.1\end{pmatrix}$$

$$\|\boldsymbol{g}(\boldsymbol{x}(t),t)\|\leqslant\alpha\|\boldsymbol{x}(t)\|$$

$$\|\boldsymbol{h}(\boldsymbol{x}(t-\tau(t)),t)\|\leqslant\beta\|\boldsymbol{x}(t-\tau(t))\|$$

$$\|\boldsymbol{k}(\dot{\boldsymbol{x}}(t-d(t)),t)\|\leqslant\gamma\|\dot{\boldsymbol{x}}(t-d(t))\|$$

$$\alpha\geqslant0,\beta\geqslant0,\gamma\geqslant0$$

表 4、5 给出了用本文定理方法计算时滞上界与相关文献计算结果的比较.

表4 时滞上界计算结果比较($\sigma=0$, $\alpha=0.1$, $\beta=0.05$, $\gamma=0.05$)

Tab.4 Comparison of calculating results of delay upper bounds ($\sigma=0$, $\alpha=0.1$, $\beta=0.05$, $\gamma=0.05$)

τ_l	$\tau_u(\sigma=0)$			$\tau_u(\sigma=0.5)$		
	文献[19]	文献[8]	本文定理	文献[19]	文献[8]	本文定理
0	1.884 2	2.207 5	3.599 9	1.690 4	1.973 4	3.424 9
0.5	2.303 2	—	4.106 2	2.109 4	—	3.718 9
1	2.803 2	—	4.602 9	2.609 4	—	4.218 9
10	11.803 2	—	13.606 4	11.609 4	—	13.218 9
100	101.803 2	—	103.606 4	101.609 4	—	103.218 9
1 000	1 001.803 2	—	1 003.606 3	1 001.609 4	—	1 003.218 9

表5 时滞上界计算结果比较($\alpha=0.3$, $\beta=0.1$, $\gamma=0.5$, $\mu=0.9$)

Tab.5 Comparison of calculating results of delay upper bounds ($\alpha=0.3$, $\beta=0.1$, $\gamma=0.5$, $\mu=0.9$)

τ_l	$\tau_u(\sigma=0)$		$\tau_u(\sigma=0.5)$	
	文献[19]	本文定理	文献[19]	本文定理
0.5	1.153 3	1.806 7	0.635 9	0.866 6
1	1.653 3	2.306 7	1.135 8	1.366 6
3	3.653 3	4.306 7	3.135 9	3.366 6
5	5.653 3	6.306 7	5.135 9	5.366 6

表4、5的结果表明,本文方法较文献[19]判据的保守性更小.

4 结 语

本文讨论了具有时变时滞和非线性扰动的中立系统稳定性分析问题.基于Lyapunov-Krasovskii泛函方法、线性矩阵不等式技术及本文提出的积分等式,得到了时滞相关稳定性判据.数值例子说明,本文的判据保守性更小.

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Delay-dependent robust stability analysis of neutral systems based on integral equalities

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Abstract: The delay-dependent robust stability criteria for neutral systems with mixed time-varying interval delays and nonlinear parameter perturbations are discussed. Based on the Lyapunov-Krasovskii functional approach and linear matrix inequality technology, some improved delay-dependent stability conditions are proposed by utilizing new integral equalities to deal with the bounding for the cross term. The criteria are derived in terms of linear matrix inequalities (LMIs). Compared to the existing criteria, the merit of the proposed conditions is less conservative. Numerical examples are given to illustrate the improvement on the conservatism of the delay bound over some reported results in the literatures.

Key words: neutral systems; LMIs; integral equality; nonlinear parameter perturbations