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Teleportation of a bipartite quantum state via generalized Bell states

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Abstract: Two-qubit quantum teleportation is closely related to quantum computation, so a teleportation protocol in which an arbitrary bipartite quantum state is perfectly teleported probabilistically from sender to receiver is proposed. One of 16 generalized non-maximally entangled Bell states (G states for simplicity) functions as quantum channel. The teleportation can be successfully realized with a certain probability if sender performs generalized Bell state measurements (G measurements) and receiver introduces an auxiliary particle and operates appropriate unitary transformations and single-qubit measurements. The probability of successful teleportation is determined by the smallest one among the coefficients' absolute values of the quantum channel.

Key words: teleportation; generalized Bell states; auxiliary particle; unitary transformations

0 Introduction

Quantum teleportation process, originally proposed by Bennett, et al. [1], can transmit an unknown quantum state from a sender to a receiver at a distant location via a quantum channel with the aid of some classical information. No information about the unknown state is ever revealed during the teleportation process. Entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure (the entangled state itself). Lots of attention were paid to both in theory and experiments in recent years [2-16]. Teleportation has been demonstrated for the polarization state of a photon [10], the state of a

trapped ion^[11,12] and, in the continuous variable regime, for the set of coherent states of a single-mode radiation field^[13]. Especially, a quantum teleportation network for continuous variables was proposed by van Loock, *et al*. ^[14] in 2000, and Yonezawa, *et al*. experimentally demonstrated tripartite quantum teleportation network for quantum states of continuous variables (electromagnetic field modes)^[15]. As for the teleportation of bipartite entanglement, it was already demonstrated for continuous variables again by Yonezawa, *et al*. ^[16].

Quantum teleportation of two qubits has recently been studied for pure and noisy quantum channels^[17-19]. It is closely related to

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quantum computation as two-qubit teleportation together with one-qubit unitary operations is sufficient to implement the universal gates required for quantum computation^[20]. For teleporting the quantum state, a number of quantum channels are used, such as the Einstein-Podolsky-Rosen (EPR) pair of twoparticle^[1], the GHZ state of three particles^[21] and the squeezed vacuum states^[22]. In Lit. [23] an arbitrary two-qubit state is deterministically teleported via 16 generalized maximally entangled Bell states. Without losing generality, the attention will be paid to teleporting an arbitrary two-qubit entangled state in this paper by using one of 16 non-maximally entangled G states as quantum channel. The scheme requires G measurements at the sender's location and appropriate unitary transformations, auxiliary particle, single-qubit measurements at the receiver's side. In the scheme, an unknown bipartite quantum state of two particles can be perfectly teleported probabilistically.

1 Teleportation of an arbitrary two-qubit entangled state

Suppose that sender Alice wants to transmit an arbitrary bipartite quantum state $|\phi\rangle_{12}$ to receiver Bob, both of whom are spatially separated. $|\phi\rangle_{12}$ can be expressed as $|\phi\rangle_{12} = (x \mid 00\rangle + y \mid 01\rangle + z \mid 10\rangle + r \mid 11\rangle)_{12}$

where x, y, z and r are unknown except that $|x|^2 + |y|^2 + |z|^2 + |r|^2 = 1$.

Now use one of 16 non-maximally entangled G state as quantum channel. The 16 non-maximally entangled G states can be divided into the following four groups:

Group 1 $|g'_{1}\rangle = a \mid 0000\rangle + b \mid 0101\rangle + c \mid 1010\rangle + d \mid 1111\rangle$ (2) $|g'_{2}\rangle = a \mid 0000\rangle + b \mid 0101\rangle - c \mid 1010\rangle - d \mid 1111\rangle$ (3)

 $\mid g_3' \rangle = a \mid 0000 \rangle - b \mid 0101 \rangle +$ $c \mid 1010 \rangle - d \mid 1111 \rangle$ (4) $\mid g_{4}' \rangle = a \mid 0000 \rangle - b \mid 0101 \rangle$ $c \mid 1010 \rangle + d \mid 1111 \rangle$ (5) Group 2 $|g_{5}'\rangle = a |0001\rangle + b |0100\rangle +$ $c \mid 1011\rangle + d \mid 1110\rangle$ (6) $|g_{6}'\rangle = a |0001\rangle + b |0100\rangle$ $c \mid 1011\rangle - d \mid 1110\rangle$ (7) $\mid g_7' \rangle = a \mid 0001 \rangle - b \mid 0100 \rangle +$ $c \mid 1011\rangle - d \mid 1110\rangle$ (8) $\mid g_{8}' \rangle = a \mid 0001 \rangle - b \mid 0100 \rangle$ $c \mid 1011\rangle + d \mid 1110\rangle$ (9) Group 3 $|g_{9}'\rangle = a |0010\rangle + b |0111\rangle +$ $c \mid 1000\rangle + d \mid 1101\rangle$ (10) $|g'_{10}\rangle = a |0010\rangle + b |0111\rangle$ $c \mid 1000\rangle - d \mid 1101\rangle$ (11) $\mid g'_{11} \rangle = a \mid 0010 \rangle - b \mid 0111 \rangle +$ $c \mid 1000\rangle - d \mid 1101\rangle$ (12) $|g'_{12}\rangle = a \mid 0010\rangle - b \mid 0111\rangle$ $c \mid 1000\rangle + d \mid 1101\rangle$ (13)Group 4 $|g'_{13}\rangle = a |0011\rangle + b |0110\rangle +$ $c \mid 1001\rangle + d \mid 1100\rangle$ (14) $|g'_{14}\rangle = a |0011\rangle + b |0110\rangle$ $c \mid 1001\rangle - d \mid 1100\rangle$ (15) $\mid g_{15}' \rangle = a \mid 0011 \rangle - b \mid 0110 \rangle +$ $c \mid 1001\rangle - d \mid 1100\rangle$ (16) $\mid g'_{16} \rangle = a \mid 0011 \rangle - b \mid 0110 \rangle$ $c \mid 1001\rangle + d \mid 1100\rangle$ (17)

where the coefficients a, b, c and d are real, $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. Since a, b, c and d are unknown probabilistic amplitudes of these states, the simplest situation is |a| = |b| = |c| = |d| = 1/2, corresponding to maximally entangled G states. Generally speaking, the coefficients a, b, c and d are different, and $|a| \neq |b| \neq |c| \neq |d|$. Without losing generality, assume $|a| \leq |b| \leq |c| \leq |d|$. For other combinations of the inequality,

similar results can be obtained.

Particles 3,4,5,6 are in one of the 16 non-maximally entangled G states. Particles 3,4 are with Alice and the last two 5,6 with Bob. For definiteness, assume Alice and Bob share state $|g_1\rangle_{3456}$. Hence, the initial joint state is $|\hspace{.06cm} \psi \rangle_{123456} = |\hspace{.06cm} \phi \rangle_{12} \otimes |\hspace{.06cm} g_1' \rangle_{3456} =$ $(x \mid 00\rangle + y \mid 01\rangle + z \mid 10\rangle +$ $r \mid 11\rangle)_{12}(a \mid 0000\rangle +$ $b \mid 0101\rangle + c \mid 1010\rangle + d \mid 1111\rangle)_{3456} =$ $(xa \mid 000000) + xb \mid 000101) +$ $xc \mid 001010\rangle + xd \mid 001111\rangle +$ $ya \mid 010000\rangle + yb \mid 010101\rangle +$ $x \mid 011010 \rangle + xd \mid 0111111 \rangle +$ $za \mid 1000000 \rangle + zb \mid 1001010 \rangle +$ $x \mid 101010\rangle + xd \mid 1011111\rangle +$ $ra \mid 110000\rangle + rb \mid 110101\rangle +$ $rc \mid 111010\rangle + rd \mid 1111111\rangle)_{123456}$ (18)

Alice makes G measurements on particles 1, 2,3 and 4, obtaining with equal probabilities one of the 16 maximally entangled G states. Then she sends Bob a classical message to inform him which maximally entangled G state she obtained. With Alice's classical information Bob knows the joint state in Eq. (18) will collapse into which state. All possible 16 collapsed states are written as the following:

$$\begin{array}{c} {}_{1234}\langle g_1 \mid \psi \rangle_{123456} = \frac{1}{2}(xa \mid 00\rangle + yb \mid 01\rangle + \\ & x \mid 10\rangle + rd \mid 11\rangle)_{56} \quad (19) \\ {}_{1234}\langle g_2 \mid \psi \rangle_{123456} = \frac{1}{2}(xa \mid 00\rangle + yb \mid 01\rangle - \\ & x \mid 10\rangle - rd \mid 11\rangle)_{56} \quad (20) \\ {}_{1234}\langle g_3 \mid \psi \rangle_{123456} = \frac{1}{2}(xa \mid 00\rangle - yb \mid 01\rangle + \\ & x \mid 10\rangle - rd \mid 11\rangle)_{56} \quad (21) \\ {}_{1234}\langle g_4 \mid \psi \rangle_{123456} = \frac{1}{2}(xa \mid 00\rangle - yb \mid 01\rangle - \\ & x \mid 10\rangle + rd \mid 11\rangle)_{56} \quad (22) \\ {}_{1234}\langle g_5 \mid \psi \rangle_{123456} = \frac{1}{2}(xb \mid 01\rangle + ya \mid 00\rangle + \\ & xd \mid 11\rangle + rc \mid 10\rangle)_{56} \quad (23) \\ {}_{1234}\langle g_6 \mid \psi \rangle_{123456} = \frac{1}{2}(xb \mid 01\rangle + ya \mid 00\rangle - \\ \end{array}$$

 $zd \mid 11\rangle - rc \mid 10\rangle)_{56}$ (24)

According to Alice's measured outcome, Bob will perform appropriate unitary transformation U_1 on particles 5,6 in order to establish a correspondence so that the coefficients x, y, z and r can correspond to $|00\rangle_{56}$, $|01\rangle_{56}$, $|10\rangle_{56}$ and $|11\rangle_{56}$, respectively.

Suppose that Alice's measured outcome is $\mid g_1 \rangle_{1234}$, so the collapsed state is in Eq. (19). After receiving Alice's classical information, Bob introduces an auxiliary two-state particle 7 with the initial state $\mid 0 \rangle_7$. In order for Bob to reincarnate the original state under the basis $\{\mid 000\rangle_{567}, \quad \mid 010\rangle_{567}, \quad \mid 100\rangle_{567}, \quad \mid 110\rangle_{567}, \quad \mid 111\rangle_{567}, \quad \mid 001\rangle_{567}, \quad \mid 011\rangle_{567}, \quad \mid 101\rangle_{567}, \quad \mid 111\rangle_{567}, \quad \mid 111\rangle_{567}, \quad \mid 101\rangle_{567}, \quad \mid 10$

5,6,7 may take the form of the following 8×8 matrix, namely

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix} \tag{35}$$

where \mathbf{A}_1 and \mathbf{A}_2 are 4×4 matrixes and may be written as

$$\mathbf{A}_1 = \mathbf{diag}\{a_0, a_1, a_2, a_3\}$$
 (36)

and

$$\mathbf{A}_{2} = \operatorname{diag}\{\sqrt{1 - a_{0}^{2}}, \sqrt{1 - a_{1}^{2}}, \sqrt{1 - a_{2}^{2}}, \sqrt{1 - a_{3}^{2}}\}$$
 (37)

respectively. Here a_i (i = 0, 1, 2, 3 and $|a_i| \le 1$) depend on the collapsed state of particles 5,6.

If choose

$$a_0=1$$
, $a_1=a/b$, $a_2=a/c$, $a_3=a/d$ (38) the unitary transformation U_2 will transform the state $_{1234}\langle g_1\mid \psi\rangle_{123456}\otimes \mid 0\rangle_7$ of particles 5,6,7 into

$$\frac{1}{2}a(x \mid 00) + y \mid 01\rangle + z \mid 10\rangle + r \mid 11\rangle)_{56} \mid 0\rangle_{7} + \frac{1}{2}(y \sqrt{b^{2} - a^{2}} \mid 01\rangle + z \sqrt{c^{2} - a^{2}} \mid 10\rangle +$$

$$r \sqrt{d^2 - a^2} \mid 11\rangle)_{56} \mid 1\rangle_7$$

Then Bob measures particle 7 on the basis of $\{ \mid 0 \rangle, \mid 1 \rangle \}$. If his measured outcome is

 $|1\rangle_6$, the teleportation fails. If $|0\rangle_6$ is obtained, the state will be collapsed into

$$(x\mid 00\rangle + y\mid 01\rangle + z\mid 10\rangle + r\mid 11\rangle)_{56}$$
 and the teleportation is successful.

A quantum channel of mixed states can never provide teleportation with fidelity 1. The probability of successful teleportation in this scheme may be expressed by the probabilistic amplitude of $\frac{1}{2}a(x\mid 00\rangle + y\mid 01\rangle + z\mid 10\rangle + r\mid 11\rangle\rangle_{56}$ as $\left(\frac{1}{2}a\right)^2 = \frac{1}{4}\mid a\mid^2$, where $\mid a\mid$ is the smallest one among the four coefficients' absolute values $\mid a\mid$, $\mid b\mid$, $\mid c\mid$ and $\mid d\mid$. Adding up all the contributions, the optimal probability of successful teleportation is obtained as $P=\frac{1}{4}\mid a\mid^2\times 16=4\mid a\mid^2$.

For the maximally entangled quantum channel, |a| = |b| = |c| = |d| = 1/2, the total successful probability reaches one.

The corresponding relations between Alice's outcomes, unitary transformation U_1 and coefficients a_i (i=0,1,2,3) of the unitary transformation U_2 are given in Tab. 1.

Tab. 1 The corresponding relations between Alice's outcomes, U_1 and coefficients of U_2

Alice's outcomes	U_1	a_0	a_1	a_2	a_3
$\mid g_1 \rangle$		1	a/b	a/c	a/d
$\mid g_2 angle$		1	a/b	-a/c	-a/d
$\mid g_3 angle$		1	-a/b	a/c	-a/d
$\mid g_4 angle$		1	-a/b	a/c	-a/d
$\mid g_5 angle$		a/b	1	a/d	a/c
$\mid g_6 angle$		a/b	-1	a/d	-a/c
$\mid g_7 angle$		a/b	1	-a/d	-a/c
$\mid g_8 angle$		a/b	-1	-a/d	a/c
$\mid g_{9} angle$		a/c	a/d	1	a/b
$\mid g_{10} angle$		a/c	-a/d	1	-a/b
$\mid g_{11} angle$		a/c	a/d	-1	-a/b
$\mid g_{12} angle$		a/c	-a/d	-1	a/b
$\mid g_{13} angle$		a/d	a/c	a/b	1
$\mid g_{14} angle$		a/d	-a/c	a/b	-1
$\mid g_{15} angle$		a/d	a/c	-a/b	-1
$\mid g_{16} angle$		a/d	-a/c	-a/b	1

When one of the other 15 non-maximally entangled G states functions as quantum channel, the teleportation process is similar.

2 Conclusions

to probabilistically realize scheme quantum teleportation of an arbitrary bipartite quantum state is proposed by using one of non-maximally entangled G states as quantum channel in this paper. It is shown that, for such non-maximally entangled quantum channel, the teleportation can be successfully realized with a certain probability by sender's G measurements receiver's appropriate unitary transformations, introducing auxiliary an particle and single-qubit measurements. It is found that the probability of success is P = $4 \mid a \mid^2$, determined by the smallest one among the four coefficients' absolute values |a|, |b|, |c| and |d|. When |a| = |b| = |c| = |d| =

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1/2, the total probability equals one.

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借助于推广 Bell 态实现两体量子态隐形传态

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摘要:两比特量子隐形传态的实现直接关系到量子计算机的实现,因此提出一个任意两比特量子态的隐形传态方案,发送者能成功地将此量子态几率地传送给接收者.此方案中,16个推广的非最大纠缠 Bell 态(简称 G 态)之一充当量子信道.发送者通过实行推广的 Bell 态测量(G 态测量),接收者通过引入一个辅助粒子并实施适当的么正变换和单粒子测量,能将此任意两比特量子态以一定的几率发送给接收者.此种隐形传态方案的成功几率由量子信道系数绝对值的最小值所决定.

关键词: 隐形传态;推广 Bell 态;辅助粒子;么正变换

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