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Stability analysis of multi-time delay systems with controller failures based on switching approach

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Abstract: The issue of robustly exponential stability for a class of uncertain multiple time-varying delay systems with controller failures and nonlinear perturbations is considered. A switched uncertain multi-delay system model is utilized to describe the considered systems. A switching approach based on the average dwell time is employed for the switched system. A delay-dependent sufficient condition for robustly exponential stability of the switched system is established in terms of linear matrix inequalities by using the switching method. It is proved theoretically that the resulting closed-loop system is robustly exponentially stable even if controller failures are encountered. The effectiveness of the proposed method is also demonstrated by simulation example.

Key words: robustly exponential stability; controller failures; average dwell time; switched systems; linear matrix inequalities (LMIs); multiple time-varying delays

0 Introduction

In some practical systems, it is frequently met to stabilize an open-loop unstable system. The controller inevitably fails over some time intervals due to some known or unpredictive cases. Therefore, the controller failure problem is expected to be a more important topic. In Lit. [1], a methodology for the design of state feedback control is presented so that the closed-loop system remains stable even when some parts of the controllers fail. In Lit. [2], using a two-channel decentralized controller configuration, necessary and sufficient conditions are obtained for the existence of reliable controllers that maintain stability under possible failure of either of the two controllers for linear MIMO plants. According to the complete breakdown of the control signal ($u(t)=0$), some results on linear time-invariant systems have been obtained based on the average dwell time method in Lit. [3,4]. But the papers mentioned above did not discuss the effect of delay.

Time-delays and perturbations are often encountered in practical control systems. Both of them are generally regarded as main sources of instability and poor performance of a control system. Considerable attention has been paid to the stability analysis and the controller design of the time-delay system[5-8]. Very recently, the time-delay systems have been extended to the switched systems[9-15]. In most of these papers, all

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subsystems are either stable or unstable. Cases are frequently encountered where unstable subsystems have to be dealt with in some practice control systems, as is described in Lit. [16,17]. In Lit. [18], a class of discrete constant time-delay switched systems with controller failure is considered by using average dwell time technique. However, no results are now available for such complex switched uncertain multi-time delay systems resulting from controller failures.

In this paper, robustly exponential stability analysis for a class of uncertain systems with multiple time-varying delays and both structure uncertainties and nonlinear perturbations is developed, whose controller fails from time to time due to physical or purposeful cases. The uncertain multi-time delay system subjected to controller failure is firstly modeled as a switched uncertain multi-time delay system including an unstable subsystem. By using the average dwell time approach combined with the integral inequality, under the conditions that the total activation time ratio of unstable subsystems to stable ones is upper bounded, with the help of the lemmas and a special piecewise Lyapunov function, a delay-dependent sufficient condition for exponential stability of the switched system is derived in terms of linear matrix inequalities. Finally, the effectiveness of the proposed method is demonstrated by simulation example.

1 Problem formulation and preliminaries

Consider the following uncertain linear system with multiple time-varying delays and nonlinear perturbations

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = {} & (\boldsymbol{A}+\Delta\boldsymbol{A}(t))\boldsymbol{x}(t)+\sum_{i=1}^{m}(\boldsymbol{A}_{\rm d}^{i}+\\ & \Delta\boldsymbol{A}_{\rm d}^{i}(t))\boldsymbol{x}(t-\tau_i(t))+f(t,\boldsymbol{x}(t))+\\ & \sum_{i=1}^{m}g_i(t,\boldsymbol{x}(t-\tau_i(t)))+\boldsymbol{Bu}(t),\\ \boldsymbol{x}(t) = {} & \boldsymbol{\varphi}(t),\ t\in[-\tau,0]\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)\in\boldsymbol{R}^n$ is the state vector and $\boldsymbol{u}(t)\in\boldsymbol{R}^m$ is the control input; $\boldsymbol{A}$, $\boldsymbol{A}_{\rm d}^{i}(i=1,2,\cdots,m)$ and $\boldsymbol{B}$ are constant matrices of appropriate dimensions; the uncertainties $\Delta\boldsymbol{A}(t)$ and $\Delta\boldsymbol{A}_{\rm d}^{i}(t)$ $(i=1,2,\cdots,m)$ are some perturbations with appropriate dimensions, and have the following forms:

$$(\Delta\boldsymbol{A}(t)\quad \Delta\boldsymbol{A}_{\rm d}^{i}(t))=\boldsymbol{DF}(t)(\boldsymbol{E}\quad \boldsymbol{E}_i);\quad i=1,2,\cdots,m$$

with $\boldsymbol{F}^{\rm T}(t)\boldsymbol{F}(t)\leqslant\boldsymbol{I}$, where $\boldsymbol{D}$, $\boldsymbol{E}$ and $\boldsymbol{E}_i$ are known constant matrices with appropriate dimensions. The functions $f(t,\boldsymbol{x}(t))$ and $g_i(t,\boldsymbol{x}(t-\tau_i(t)))$ $(i=1,2,\cdots,m)$ are unknown nonlinear uncertainties. It is assumed that $f(t,\boldsymbol{0})=\boldsymbol{0}$, $g_i(t,\boldsymbol{0})=\boldsymbol{0}(i=1,2,\cdots,m)$ and

$$\begin{aligned}&\boldsymbol{f}^{\rm T}\boldsymbol{f}\leqslant a^2\boldsymbol{x}^{\rm T}(t)\boldsymbol{x}(t)\\ &\boldsymbol{g}_i^{\rm T}\boldsymbol{g}_i\leqslant b_i^2\boldsymbol{x}^{\rm T}(t-\tau_i(t))\boldsymbol{x}(x-\tau_i(t))\end{aligned} \tag{2}$$

where, for simplicity, $f:f(t,\boldsymbol{x}(t))$, $g_i:g_i(t,\boldsymbol{x}(t-\tau_i(t)))$, $\tau_i(t)$ $(i=1,2,\cdots,m)$ are the time-varying delay of the system, and satisfy one of the following conditions:

(H1) $0\leqslant\tau_i(t)\leqslant\tau_i$, $\dot{\tau}_i(t)\leqslant\mu_i$ with $\tau_i>0$;

(H2) $0\leqslant\tau_i(t)\leqslant\tau_i$ with $\tau_i>0$.

$\boldsymbol{\varphi}(t)$ is a continuous initial function on $[-\tau,0]$ with $\tau=\max\limits_{1\leqslant i\leqslant m}\{\tau_i\}$. Throughout this paper, it is assumed that: (1) $\boldsymbol{A}$ is unstable; (2) $(\boldsymbol{A},\boldsymbol{B})$ is stabilizable; (3) a state feedback controller $\boldsymbol{u}(t)=\boldsymbol{Kx}(t)$ has been designed such that $\boldsymbol{A}_{\rm s}=\boldsymbol{A}+\boldsymbol{BK}$ is stable. When the controller works, System (1) is expressed as follows:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = {} & \boldsymbol{A}_{\rm s}(t)\boldsymbol{x}(t)+\sum_{i=1}^{m}\boldsymbol{A}_{\rm d}^{i}(t)\boldsymbol{x}(t-\tau_i(t))+\\ & f(t,\boldsymbol{x}(t))+\sum_{i=1}^{m}g_i(t,\boldsymbol{x}(t-\tau_i(t))),\\ \boldsymbol{x}(t) = {} & \boldsymbol{\varphi}(t),\ t\in[-\tau,0]\end{aligned} \tag{3}$$

where $\boldsymbol{A}_{\rm s}(t)=\boldsymbol{A}_{\rm s}+\Delta\boldsymbol{A}(t)$, $\boldsymbol{A}_{\rm d}^{i}(t)=\boldsymbol{A}_{\rm d}^{i}+\Delta\boldsymbol{A}_{\rm d}^{i}(t)$. When the controller fails completely, System (1) is rewritten as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = {} & \boldsymbol{A}(t)\boldsymbol{x}(t)+\sum_{i=1}^{m}\boldsymbol{A}_{\rm d}^{i}(t)\boldsymbol{x}(t-\tau_i(t))+\\ & f(t,\boldsymbol{x}(t))+\sum_{i=1}^{m}g_i(t,\boldsymbol{x}(t-\tau_i(t))),\\ \boldsymbol{x}(t) = {} & \boldsymbol{\varphi}(t),\ t\in[-\tau,0]\end{aligned} \tag{4}$$

Where $\boldsymbol{A}(t)=\boldsymbol{A}+\Delta\boldsymbol{A}(t)$. Then, the entire system dynamics can be expressed as a switched system composed of an unstable subsystem and a stable subsystem. The form is expressed as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = {} & \boldsymbol{A}_{\sigma(t)}(t)\boldsymbol{x}(t)+\sum_{i=1}^{m}\boldsymbol{A}_{\rm d}^{i}(t)\boldsymbol{x}(t-\tau_i(t))+\\ & f(t,\boldsymbol{x}(t))+\sum_{i=1}^{m}g_i(t,\boldsymbol{x}(t-\tau_i(t))),\end{aligned}$$

$$x(t)=\varphi(t),\ t\in[-\tau,0] \tag{5}$$

Where $\sigma(t):[0,+\infty)\to \underline{P}=\{1,2\}$, $A_1(t)=A_s(t)$, $A_2(t)=A(t)$. $\sigma(t)=1$ means the controller works; $\sigma(t)=2$ means the controller fails.

The objective of this paper is to design a class of switching laws under which System (5) is exponentially stable. To formulate the problem, some definitions and lemmas are introduced.

Definition 1[19] For each switching law σ and each $T\geqslant t\geqslant 0$, let $N_\sigma(T,t)$ denote the number of switching of σ over the interval (t,T). If

$$N_\sigma(T,t)\leqslant\frac{T-t}{\tau_a},\ \forall\,\tau_a>0 \tag{6}$$

τ_a is called the average dwell time.

Definition 2 System (5) is said to be exponentially stable if there exist scalars $\lambda>0$, $\kappa>1$ such that for all $x(t)$ the following inequality holds:

$$\|x(t)\|\leqslant\kappa\|x_{t_0}\|_{cl}e^{-\lambda(t-t_0)},\ \forall\,t\geqslant t_0 \tag{7}$$

where $\|x_t\|_{cl}:=\sup_{-\tau\leqslant\theta\leqslant 0}\{\|x(t+\theta)\|,\ \|\dot{x}(t+\theta)\|\}$, $\|\cdot\|$ is the Euclidean norm.

Lemma 1[20] For any constant symmetric matrix $W>0$, scalar $\tau>0$, and vector function $\dot{x}(\cdot):[-\tau,0]\to R^n$ such that the following integral is well defined, then

$$-\tau\int_{t-\tau}^{t}\dot{x}^{T}(s)W\dot{x}(s)\mathrm{d}s\leqslant z^{T}(t)\begin{pmatrix}-W & W\\ W & -W\end{pmatrix}z(t) \tag{8}$$

where $z(t)=(x^{T}(t)\quad x^{T}(t-\tau))^{T}$.

Lemma 2[21] Let D, F, E and M be real matrices of appropriate dimensions with M satisfying $M=M^T$, then for all $F^TF\leqslant I$, the fact that $M+DFE+E^TF^TD^T<0$ holds if and only if there exists $\varepsilon>0$ such that $M+\varepsilon^{-1}DD^T+\varepsilon E^TE<0$.

2 Main results

Due to System (3), the matrix $A_s=A+BK$ is stable, so a scalar $\lambda^->0$ can be obtained such that $A_s+\lambda^- I$ is stable.

Lemma 3 Under (H1), for given $a>0$, $b_i>0$, $\lambda^->0$ and for allowable upper bounds $\tau_i>0$, if there exist positive symmetric matrices P_1, Q_i^1, R_i^1 and positive scalars ε_1, δ_1, η_i^1 $(i=1,2,\cdots,m)$ such that the following linear matrix inequality holds

$$\begin{pmatrix}\Pi_{11}^1+\delta_1\Delta & \Pi_{12}^1 & \Pi_{13}^1 & \Pi_{14}^1\\ \Pi_{12}^1 & \Pi_{22}^1 & \Pi_{23}^1 & \Pi_{24}^1\\ \Pi_{13}^1 & \Pi_{23}^1 & \Pi_{33}^1 & \Pi_{34}^1\\ \Pi_{14}^1 & \Pi_{24}^1 & \Pi_{34}^1 & \Pi_{44}^1\end{pmatrix}<0 \tag{9}$$

where

$$\Pi_{11}^1=\begin{pmatrix}\phi_{11}^1 & \phi_{12}^1 & \phi_{13}^1 & \cdots & \phi_{1m+1}^1\\ \phi_{12}^1 & \phi_{22}^1 & \phi_{23}^1 & \cdots & \phi_{2m+1}^1\\ \phi_{13}^1 & \phi_{23}^1 & \phi_{33}^1 & \cdots & \phi_{3m+1}^1\\ \vdots & \vdots & \vdots & & \vdots\\ \phi_{1m+1}^1 & \phi_{2m+1}^1 & \phi_{3m+1}^1 & \cdots & \phi_{m+1m+1}^1\end{pmatrix}$$

with

$$\phi_{11}^1=P_1(A_s+\lambda^- I)+(A_s+\lambda^- I)^TP_1+\sum_{i=1}^{m}Q_i^1-\sum_{i=1}^{m}\tau_i^{-1}R_i^1e^{-2\lambda^-\tau_i}+\varepsilon_1a^2I,$$

$$\phi_{12}^1=P_1A_d^1+\tau_1^{-1}R_1^1e^{-2\lambda^-\tau_1},\cdots,\ \phi_{1m+1}^1=P_1A_d^m+\tau_m^{-1}R_m^1e^{-2\lambda^-\tau_m},$$

$$\phi_{22}^1=-[(1-\mu_1)Q_1^1e^{-2\lambda^-\tau_1}+\tau_1^{-1}R_1^1e^{-2\lambda^-\tau_1}]+\eta_1^1b_1^2I,$$

$$\phi_{33}^1=-[(1-\mu_2)Q_2^1e^{-2\lambda^-\tau_2}+\tau_2^{-1}R_2^1e^{-2\lambda^-\tau_2}]+\eta_2^1b_2^2I,\cdots,$$

$$\phi_{m+1m+1}^1=-[(1-\mu_m)Q_m^1e^{-2\lambda^-\tau_m}+\tau_m^{-1}R_m^1e^{-2\lambda^-\tau_m}]+\eta_m^1b_m^2I,$$

$\phi_{23}^1=\cdots=\phi_{2m+1}^1=\phi_{34}^1=\cdots=\phi_{3m+1}^1=\cdots=\phi_{m-1m}^1=\phi_{m-1m+1}^1=\phi_{mm+1}^1=V$ (V is a zero matrix of appropriate dimension),

$$\Pi_{12}^1=\begin{pmatrix}P_1 & P_1 & \cdots & P_1\\ 0 & 0 & \cdots & 0\\ \vdots & \vdots & & \vdots\\ 0 & 0 & \cdots & 0\end{pmatrix},\ \Pi_{14}^1=\begin{pmatrix}P_1D\\ 0\\ \vdots\\ 0\end{pmatrix},$$

$$\Pi_{24}^1=\begin{pmatrix}0\\ 0\\ \vdots\\ 0\end{pmatrix},\ \Pi_{34}^1=\begin{pmatrix}R_1^1D\\ R_2^1D\\ \vdots\\ R_m^1D\end{pmatrix},$$

$$\Pi_{13}^1=(A_s\ A_d^1\ \cdots\ A_d^m)^T(R_1^1\ \cdots\ R_m^1),$$

$$\Pi_{23}^1=(I\ \cdots\ I)^T(R_1^1\ \cdots\ R_m^1),$$

$$\Pi_{22}^1=-\mathbf{diag}\{\varepsilon_1I,\eta_1^1I,\cdots,\eta_m^1I\},$$

$$\Pi_{33}^1=-\mathbf{diag}\{\tau_1^{-1}R_1^1,\cdots,\tau_m^{-1}R_m^1\},\ \Pi_{44}^1=-\delta_1I,$$

$$\Delta=(E\ E_1\ \cdots\ E_m)^T(E\ E_1\ \cdots\ E_m)$$

Then, along the trajectory of System (3), it results in the following equation:

$$V_1(t)\leqslant e^{-2\lambda^-(t-t_0)}V_1(t_0) \tag{10}$$

Proof Choose the Lyapunov-Krasovskii functional candidate

$$\boldsymbol{V}_1(t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_1\boldsymbol{x}(t)+\sum_{i=1}^{m}\int_{t-\tau_i(t)}^{t}(\boldsymbol{x}^{\mathrm{T}}(s)\times \boldsymbol{Q}_i^1\mathrm{e}^{2\lambda^-(s-t)}\boldsymbol{x}(s))\mathrm{d}s+\sum_{i=1}^{m}\int_{-\tau_i}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{T}(s)\boldsymbol{R}_i^1\mathrm{e}^{2\lambda^-(s-t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{11}$$

Then along the trajectory of System (3) and by Lemma 1 together with Eq. (2), it leads to

$$\dot{\boldsymbol{V}}_1(t)+2\lambda^-\boldsymbol{V}_1(t)\leqslant\boldsymbol{\zeta}^{\mathrm{T}}(t)\boldsymbol{\Pi}_1(t)\boldsymbol{\zeta}(t)$$

where

$$\boldsymbol{\Pi}_1(t)=\begin{pmatrix}\boldsymbol{\Pi}_{11}^1(t) & \boldsymbol{\Pi}_{12}^1\\ \boldsymbol{\Pi}_{12}^{1\mathrm{T}}(t) & \boldsymbol{\Pi}_{22}^1\end{pmatrix}+\boldsymbol{\Lambda}_1^{\mathrm{T}}(t)\sum_{i=1}^{m}\tau_i\boldsymbol{R}_i^1\boldsymbol{\Lambda}_1(t),$$

$$\boldsymbol{\Lambda}_1(t)=(\boldsymbol{A}_s(t)\ \boldsymbol{A}_d^1(t)\ \cdots\ \boldsymbol{A}_d^m(t)\ \boldsymbol{I}\ \boldsymbol{I}\ \cdots\ \boldsymbol{I})$$

By Schur complements and Lemma 2, from Eq. (9) it is easy to see that $\boldsymbol{\Pi}_1(t)<\boldsymbol{0}$. Thus $\dot{\boldsymbol{V}}_1(t)+2\lambda^-\boldsymbol{V}_1(t)\leqslant\boldsymbol{0}$.

Integrating the above inequality, it obviously holds that

$$\boldsymbol{V}_1(t)\leqslant\mathrm{e}^{-2\lambda^-(t-t_0)}\boldsymbol{V}_1(t_0) \tag{12}$$

When the controller fails, System (1) reduces to System (4). As is well known, matrix $\boldsymbol{A}$ is unstable. There exists a constant $\lambda^+>0$ such that $\boldsymbol{A}-\lambda^+\boldsymbol{I}$ is stable. The following resuls may be obtained.

Lemma 4 Under (H1), for given $a>0$, $b_i>0$, $\lambda^+>0$ and for allowable upper bounds $\tau_i>0$, if there exist positive symmetric matrices $\boldsymbol{P}_2$, $\boldsymbol{Q}_i^2$, $\boldsymbol{R}_i^2$ and positive scalars ε_2, δ_2, $\eta_i^2(i=1,2,\cdots,m)$ such that the following linear matrix inequality holds

$$\begin{pmatrix}\boldsymbol{\Pi}_{11}^2+\delta_2\boldsymbol{\Delta} & \boldsymbol{\Pi}_{12}^2 & \boldsymbol{\Pi}_{13}^2 & \boldsymbol{\Pi}_{14}^2\\ \boldsymbol{\Pi}_{12}^2 & \boldsymbol{\Pi}_{22}^2 & \boldsymbol{\Pi}_{23}^2 & \boldsymbol{\Pi}_{24}^2\\ \boldsymbol{\Pi}_{13}^2 & \boldsymbol{\Pi}_{23}^2 & \boldsymbol{\Pi}_{33}^2 & \boldsymbol{\Pi}_{34}^2\\ \boldsymbol{\Pi}_{14}^2 & \boldsymbol{\Pi}_{24}^2 & \boldsymbol{\Pi}_{34}^2 & \boldsymbol{\Pi}_{44}^2\end{pmatrix}<\boldsymbol{0} \tag{13}$$

where

$$\boldsymbol{\Pi}_{11}^2=\begin{pmatrix}\boldsymbol{\phi}_{11}^2 & \boldsymbol{\phi}_{12}^2 & \boldsymbol{\phi}_{13}^2 & \cdots & \boldsymbol{\phi}_{1m+1}^2\\ \boldsymbol{\phi}_{12}^2 & \boldsymbol{\phi}_{22}^2 & \boldsymbol{\phi}_{23}^2 & \cdots & \boldsymbol{\phi}_{2m+1}^2\\ \boldsymbol{\phi}_{13}^2 & \boldsymbol{\phi}_{23}^2 & \boldsymbol{\phi}_{33}^2 & \cdots & \boldsymbol{\phi}_{3m+1}^2\\ \vdots & \vdots & \vdots & & \vdots\\ \boldsymbol{\phi}_{1m+1}^2 & \boldsymbol{\phi}_{2m+1}^2 & \boldsymbol{\phi}_{3m+1}^2 & \cdots & \boldsymbol{\phi}_{m+1m+1}^2\end{pmatrix}$$

with

$$\boldsymbol{\phi}_{11}^2=\boldsymbol{P}_2(\boldsymbol{A}-\lambda^+\boldsymbol{I})+(\boldsymbol{A}-\lambda^+\boldsymbol{I})^{\mathrm{T}}\boldsymbol{P}_2+\sum_{i=1}^{m}\boldsymbol{Q}_i^2-\sum_{i=1}^{m}\tau_i^{-1}\boldsymbol{R}_i^2\mathrm{e}^{-2\lambda^+\tau_i}+\varepsilon_2a^2\boldsymbol{I},$$

$$\boldsymbol{\phi}_{12}^2=\boldsymbol{P}_2\boldsymbol{A}_d^1+\tau_1^{-1}\boldsymbol{R}_1^2\mathrm{e}^{-2\lambda^+\tau_1},$$

$$\boldsymbol{\phi}_{13}^2=\boldsymbol{P}_2\boldsymbol{A}_d^2+\tau_2^{-1}\boldsymbol{R}_2^2\mathrm{e}^{-2\lambda^+\tau_2},\cdots,$$

$$\boldsymbol{\phi}_{1m+1}^2=\boldsymbol{P}_2\boldsymbol{A}_d^m+\tau_m^{-1}\boldsymbol{R}_m^2\mathrm{e}^{-2\lambda^+\tau_m},$$

$$\boldsymbol{\phi}_{22}^2=-[(1-\mu_1)\boldsymbol{Q}_1^2\mathrm{e}^{-2\lambda^+\tau_1}+\tau_1^{-1}\boldsymbol{R}_1^2\mathrm{e}^{-2\lambda^+\tau_1}]+\eta_1^2b_1^2\boldsymbol{I},$$

$$\boldsymbol{\phi}_{33}^2=-[(1-\mu_2)\boldsymbol{Q}_2^2\mathrm{e}^{-2\lambda^+\tau_2}+\tau_2^{-1}\boldsymbol{R}_2^2\mathrm{e}^{-2\lambda^+\tau_2}]+\eta_2^2b_2^2\boldsymbol{I},\cdots,$$

$$\boldsymbol{\phi}_{m+1m+1}^2=-[(1-\mu_m)\boldsymbol{Q}_m^2\mathrm{e}^{-2\lambda^+\tau_m}+\tau_m^{-1}\boldsymbol{R}_m^2\mathrm{e}^{-2\lambda^+\tau_m}]+\eta_m^2b_m^2\boldsymbol{I},$$

$$\boldsymbol{\phi}_{23}^2=\cdots=\boldsymbol{\phi}_{2m+1}^2=\boldsymbol{\phi}_{34}^2=\cdots=\boldsymbol{\phi}_{3m+1}^2=\cdots=\boldsymbol{\phi}_{m-1m}^2=\boldsymbol{\phi}_{m-1m+1}^2=\boldsymbol{\phi}_{mm+1}^2=\boldsymbol{V},$$

$$\boldsymbol{\Pi}_{12}^2=\begin{pmatrix}\boldsymbol{P}_2 & \boldsymbol{P}_2 & \cdots & \boldsymbol{P}_2\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \vdots & \vdots & & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\end{pmatrix},\ \boldsymbol{\Pi}_{14}^2=\begin{pmatrix}\boldsymbol{P}_2\boldsymbol{D}\\ \boldsymbol{0}\\ \vdots\\ \boldsymbol{0}\end{pmatrix},$$

$$\boldsymbol{\Pi}_{24}^2=\begin{pmatrix}\boldsymbol{0}\\ \boldsymbol{0}\\ \vdots\\ \boldsymbol{0}\end{pmatrix},\ \boldsymbol{\Pi}_{34}^2=\begin{pmatrix}\boldsymbol{R}_1^2\boldsymbol{D}\\ \boldsymbol{R}_2^2\boldsymbol{D}\\ \vdots\\ \boldsymbol{R}_m^2\boldsymbol{D}\end{pmatrix},$$

$$\boldsymbol{\Pi}_{13}^2=(\boldsymbol{A}\quad \boldsymbol{A}_d^1\quad \cdots\boldsymbol{A}_d^m)^{\mathrm{T}}(\boldsymbol{R}_1^2\quad\cdots\quad\boldsymbol{R}_m^2),$$

$$\boldsymbol{\Pi}_{23}^2=(\boldsymbol{I}\quad\cdots\quad\boldsymbol{I})^{\mathrm{T}}(\boldsymbol{R}_1^2\quad\cdots\quad\boldsymbol{R}_m^2),$$

$$\boldsymbol{\Pi}_{22}^2=-\mathbf{diag}\{\varepsilon_2\boldsymbol{I},\eta_1^2\boldsymbol{I},\cdots,\eta_m^2\boldsymbol{I}\},$$

$$\boldsymbol{\Pi}_{33}^2=-\mathbf{diag}\{\tau_1^{-1}\boldsymbol{R}_1^2,\cdots,\tau_m^{-1}\boldsymbol{R}_m^2\},\ \boldsymbol{\Pi}_{44}^2=-\delta_2\boldsymbol{I}$$

then, along the trajectory of System (4), it has

$$\boldsymbol{V}_2(t)\leqslant\mathrm{e}^{2\lambda^+(t-t_0)}\boldsymbol{V}_2(t_0) \tag{14}$$

Proof Similar to Lemma 3, choose the Lyapunov-Krasovskii functional candidate

$$\boldsymbol{V}_2(t)=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}_2\boldsymbol{x}(t)+\sum_{i=1}^{m}\int_{t-\tau_i(t)}^{t}(\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{Q}_i^2\times \mathrm{e}^{2\lambda^+(s-t)}\boldsymbol{x}(s))\mathrm{d}s+\sum_{i=1}^{m}\int_{-\tau_i}^{0}\int_{t+\theta}^{t}(\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\times R_i^2\mathrm{e}^{2\lambda^+(s-t)}\dot{\boldsymbol{x}}(s))\mathrm{d}s\mathrm{d}\theta \tag{15}$$

From the proof of Lemma 3, one can get

$$\boldsymbol{V}_2(t)\leqslant\mathrm{e}^{2\lambda^+(t-t_0)}\boldsymbol{V}_2(t_0) \tag{16}$$

In the following section, the stability condition of switched System (5) is given based on the above two Lemmas. First, a class of switching laws is designed.

Let T_u and T_s be the total activation time of the system (when the controller fails) and the system (when the controller works) during time interval $[t_0, t]$, respectively, and choose a scalar $\lambda^* \in (\lambda, \lambda^-)$. For $\forall \lambda \in (0, \lambda^-)$, consider a class of switching laws satisfying the following two conditions[16]:

(S) $\inf\limits_{t \geqslant t_0} \dfrac{T_s}{T_u} \geqslant \dfrac{\lambda^+ + \lambda^*}{\lambda^- - \lambda^*}$, $\forall\ t \geqslant t_0$

where $\lambda^+ > 0$ and $\lambda^- > 0$ are to be chosen later.

(T) $\tau_a \geqslant \tau_a^* = \dfrac{\ln \mu}{2(\lambda^* - \lambda)}$

Then, the following theorem can be obtained.

Theorem 1 Consider switched Systems (5) satisfying (H1), for given $a > 0$, $b_i > 0$ and allowable upper bounds $\tau_i > 0$, if there exist positive symmetric matrices $\boldsymbol{P}_j$, $\boldsymbol{Q}_i^j$, $\boldsymbol{R}_i^j$ and positive scalars λ^-, λ^+, ε_j, δ_j, $\eta_i^j (j=1,2)$ $(i=1,2,\cdots,m)$ such that the following linear matrix inequalities (9) and (13) hold, then, the System (5) is robustly exponentially stable for any switching signal satisfying the conditions (S) and (T). Moreover, the state of System (5) is given by

$$\|\boldsymbol{x}(t)\| \leqslant k\mathrm{e}^{-\lambda(t-t_0)} \|\boldsymbol{x}_{t_0}\|_{\mathrm{cl}} \tag{17}$$

where

$$\mu \geqslant 1 \text{ satisfies } \boldsymbol{P}_j \leqslant \mu \boldsymbol{P}_k,\ \sum_{i=1}^m \boldsymbol{R}_i^j \leqslant \mu \sum_{i=1}^m \boldsymbol{R}_i^k,$$
$$\sum_{i=1}^m \boldsymbol{U}_i^j \leqslant \mu \sum_{i=1}^m \boldsymbol{U}_i^k,\ \forall j,k \in \underline{P},\ i = 1,2,\cdots,m \tag{18}$$

$$\lambda = \lambda^* - \frac{\ln \mu}{2\tau_a},\ k = \sqrt{\frac{\gamma}{\min\limits_{\forall j \in \underline{P}} \lambda_{\min}(\boldsymbol{P}_j)}} \geqslant 1$$

with

$$\gamma = \max_{\forall j \in \underline{P}} \lambda_{\max}(\boldsymbol{P}_j) + \tau \max_{\forall j \in \underline{P}} \lambda_{\max} \sum_{i=1}^m \boldsymbol{R}_i^j + \frac{\tau^2}{2} \max_{\forall j \in \underline{P}} \lambda_{\max} \sum_{i=1}^m \boldsymbol{U}_i^j \tag{19}$$

$\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a symmetric matrix, respectively.

Proof Choose the following piecewise Lyapunov-Krasovskii functional candidate

$$\boldsymbol{V}(t) = \boldsymbol{V}_{\sigma(t)}(t) \tag{20}$$

where $\boldsymbol{V}_1(t)$ and $\boldsymbol{V}_2(t)$ are designed in Eqs. (11) and (15).

From Eqs. (18) and (19), it is obvious that

$$\boldsymbol{V}_j(t) \leqslant \mu \boldsymbol{V}_k(t),\ \forall j,k \in \underline{P} \tag{21}$$

For $t > 0$, let $t_0 < t_1 < \cdots < t_k = t_{N_{\sigma(t_0,t)}}$ denote the switching points of σ over the interval (t_0, t). By using the differential theory and combining Eqs. (9), (13) and Lemmas 3, 4, it leads to

$$\boldsymbol{V}(t) = \boldsymbol{V}_{\sigma(t)}(t) = \boldsymbol{V}_j(t) \leqslant \begin{cases} \mathrm{e}^{-2\lambda^-(t-t_k)} \boldsymbol{V}_j(t_k),\ j = 1 \\ \mathrm{e}^{2\lambda^+(t-t_k)} \boldsymbol{V}_j(t_k),\ j = 2 \end{cases} \tag{22}$$

It follows from Conditions (S), (T), Eqs. (21) and (22) that

$$\begin{aligned} \boldsymbol{V}(t) &\leqslant \boldsymbol{V}_{\sigma(t_k)}(t_k) \mathrm{e}^{2\lambda^+ T_u(t_k,t) - 2\lambda^- T_s(t_k,t)} \leqslant \\ &\mathrm{e}^{2\lambda^+ T_u(t_k,t) - 2\lambda^- T_s(t_k,t)} \mu \boldsymbol{V}_{\sigma(t_k^-)}(t_k^-) \leqslant \cdots \leqslant \\ &\mu^{N_\sigma(t_0,t)} \mathrm{e}^{2\lambda^+ T_u(t_k,t) - 2\lambda^- T_s(t_k,t)} \boldsymbol{V}_{\sigma(t_0)}(t_0) = \\ &\mathrm{e}^{2\lambda^+ T_u(t_k,t) - 2\lambda^- T_s(t_k,t) + N_\sigma(t_0,t)\ln\mu} \boldsymbol{V}_{\sigma(t_0)}(t_0) \leqslant \\ &\mathrm{e}^{-2\lambda^*(T_u+T_s) + \frac{t-t_0}{\tau_a}\ln\mu} \boldsymbol{V}_{\sigma(t_0)}(t_0) \leqslant \\ &\mathrm{e}^{-2\left(\lambda^* - \frac{\ln\mu}{2\tau_a}\right)(t-t_0)} \boldsymbol{V}_{\sigma(t_0)}(t_0) \leqslant \\ &\mathrm{e}^{-2\lambda(t-t_0)} \boldsymbol{V}_{\sigma(t_0)}(t_0) \end{aligned} \tag{23}$$

From the definition of Lyapunov function together with Eq. (23), the following inequalities hold

$$\min_{\forall j \in \underline{P}} \lambda_{\min}(\boldsymbol{P}_j) \|\boldsymbol{x}(t)\|^2 \leqslant \boldsymbol{V}(t),$$
$$\boldsymbol{V}_{\sigma(t_0)}(t_0) \leqslant \gamma \|\boldsymbol{x}_{t_0}\|_{\mathrm{cl}}^2 \tag{24}$$

Where γ is given in Eq. (19). It follows from Eqs. (23) and (24) that

$$\|\boldsymbol{x}(t)\|^2 \leqslant \frac{\gamma}{\min\limits_{\forall j \in \underline{P}} \lambda_{\min}(\boldsymbol{P}_j)} \mathrm{e}^{-2\lambda(t-t_0)} \|\boldsymbol{x}_{t_0}\|_{\mathrm{cl}}^2$$

Therefore

$$\|\boldsymbol{x}(t)\| \leqslant k\mathrm{e}^{-\lambda(t-t_0)} \|\boldsymbol{x}_{t_0}\|_{\mathrm{cl}} \qquad \square$$

Remark 1 It is obvious to see that there exist some parameters in Theorem 1. Here some parameters, for example, scalars $a > 0$ and $b_i > 0$, are given in advance; some parameters, such as positive scalars λ^- and λ^+, are chosen with transcendental method; the upper bounds of time-delays τ_i are obtained through the following method by giving proper choice μ_i.

If the delay only satisfies (H2), by choosing $\boldsymbol{Q}_i^{\sigma(t)} = \boldsymbol{0}$ in Eqs. (9) and (13), Corollary 1 is obtained.

Corollary 1 Under (H2), for some given conditions in Theorem 1, if the following linear matrix inequalities (9) and (13) with $\boldsymbol{Q}_i^{\sigma(t)} = \boldsymbol{0}$ hold, System (5) is robustly exponentially stable for any switching signals satisfying the Conditions (S) and (T).

Remark 2 If $j = 1$ in Corollary 1, the system is non-switched uncertain system with multiple time-varying delays and nonlinear perturbations. So Corollary 1 contains the existing results in Lit. [8] as a special case.

3 Numerical example

In this section, an example is used to illustrate the effectiveness of the proposed approach.

Example 1 Consider the dynamic systems with multiple time-varying delays and both structure uncertainties and nonlinear perturbations in Systems (1), (2), Conditions (H1) and (H2) with $\boldsymbol{A}=\begin{pmatrix}0 & 1\\ -1 & 1\end{pmatrix}$, $\boldsymbol{B}=\begin{pmatrix}0\\ 1\end{pmatrix}$, $\boldsymbol{A}_{\mathrm{d}}^{1}=\begin{pmatrix}0.3 & 0.2\\ 0.2 & 0.3\end{pmatrix}$ and $\boldsymbol{A}_{\mathrm{d}}^{2}=\begin{pmatrix}0.2 & 0.1\\ 0.1 & 0.3\end{pmatrix}$; the controller is $\boldsymbol{u}=\boldsymbol{K}\boldsymbol{x}$ with $\boldsymbol{K}=(-2.3986\quad -4.4593)$, then $\boldsymbol{A}_{\mathrm{s}}=\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}=\begin{pmatrix}0 & 1\\ -3.3986 & -3.4593\end{pmatrix}$; $\boldsymbol{M}=0.01\boldsymbol{I}$, $\boldsymbol{N}=0.2\boldsymbol{I}$, $\boldsymbol{N}_{\mathrm{d}}^{1}=0.3\boldsymbol{I}$, $\boldsymbol{N}_{\mathrm{d}}^{2}=0.1\boldsymbol{I}$, $a=0.1$, $b_1=0.03$, $b_2=0.04$. It is obvious to see that $\boldsymbol{A}$ is unstable and $\boldsymbol{A}_{\mathrm{s}}$ is stable. By using Theorem 1, choose $\lambda^{+}=3$, $\lambda^{-}=0.4$, $\lambda^{*}=0.1$, $\lambda=0.05$ and $\mu=1.053$, System (5) is exponentially stable. The allowable upper bound of delays is obtained for different μ_1 and μ_2, τ_1, $\tau_2\leqslant 0.3470$ for $\mu_1=\mu_2=0$, τ_1, $\tau_2\leqslant 0.2798$ for $\mu_1=\mu_2=0.5$, τ_1, $\tau_2\leqslant 0.2785$ for $\mu_1=\mu_2=0.9$. The average dwell time is computed as $\tau_{\mathrm{a}}^{*}=\dfrac{\ln\mu}{2(\lambda^{*}-\lambda)}=0.5164$, and the switching laws require $\dfrac{T_{\mathrm{s}}}{T_{\mathrm{u}}}\geqslant\dfrac{\lambda^{+}+\lambda^{*}}{\lambda^{-}-\lambda^{*}}=10.3333$. Moreover, due to τ_1, $\tau_2\leqslant 0.2785$ for $\mu_1=\mu_2=0.9$, the state of System (5) is given by $\|\boldsymbol{x}(t)\|\leqslant 5.5661\mathrm{e}^{-0.05(t-t_0)}\|\boldsymbol{x}_{t_0}\|_{\mathrm{cl}}$.

The state trajectory of the system during all over operation is shown in Fig. 1 in which the two subsystems are activated over time periods 0.3 and 3.1 respectively, where the initial condition is $\boldsymbol{x}_0=(-5\quad 10)^{\mathrm{T}}$.

From Fig. 1, it can be seen that the system dynamics via switching are convergent quickly under the Conditions (S) and (T).

By Corollary 1, h_1, $h_2\leqslant 0.275$ is obtained if the derivative of delay is unknown.

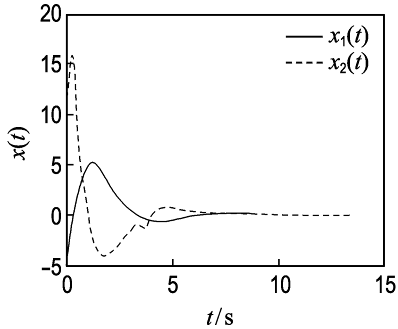

Fig. 1　The state response curves based on switching method

4 Conclusion

In this paper, robustly exponential stability analysis of switched uncertain multi-time delay systems is studied. Based on an average dwell time technique combined with linear matrix inequalities, it is shown that under a class of switching laws where the average dwell time is sufficiently large and the total activation time of the unstable subsystem is relatively small compared with that of the stable one, the switched uncertain systems with multiple time delays are robustly exponentially stable. Moreover, the derivative of the time-delay may allow to be any large or even unknown. At last, an example illustrates the effectiveness of the proposed method.

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基于切换方法的控制器故障下多时滞系统稳定性分析

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摘要：研究了控制器故障下具有非线性干扰的多时变时滞不确定系统的鲁棒指数稳定性问题.利用不确定多时变时滞切换系统模型描述了所考虑的系统.提出了基于平均驻留时间的切换方法.在此方法下，以线性矩阵不等式(LMIs)形式得到了切换系统鲁棒指数稳定的充分条件，并从理论上证明了即使控制器失效所生成的闭环系统也是鲁棒指数稳定的.最后通过仿真算例验证了所提方法的有效性.

关键词：鲁棒指数稳定性；控制器失效；平均驻留时间；切换系统；LMIs；多时变时滞

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