

一类具有混合时滞的神经网络指数稳定性分析

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摘要: 讨论了一类具有混合时滞(包含离散和分布时滞)的人工神经网络的指数稳定性问题. 通过将时滞区间分为不等的两部分, 并结合倒数凸方法, 得到了系统指数稳定的新判据, 判据以线性矩阵不等式的形式给出. 最后用两个数值实例说明了所得结论的有效性与更小的保守性.

关键词: 混合时滞; 指数稳定; 倒数凸方法; 线性矩阵不等式

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0 引言

近年来, 人工神经网络被广泛应用于信号处理、模式识别、联想记忆等诸多领域^[1-3]. 在实际应用中, 一般要求设计的神经网络模型必须是稳定的. 而时滞的存在往往导致神经网络不稳定. 因此, 时滞人工神经网络的稳定性分析已经引起了国内外众多学者的极大关注, 并得到了大量优秀的成果^[4-17].

为了降低稳定性条件的保守性, 文献[4-6]利用积分不等式或积分等式方法, 文献[7]通过构造新的 Lyapunov-Krasovskii 泛函, 文献[8-9]使用自由权矩阵的方法, 文献[10-11]运用时滞分解方法并构造新的 Lyapunov-Krasovskii 泛函, 分别得到了一系列的稳定性条件. 但是, 以上得到的结果仍有改进的空间. 如文献[10-11]将时滞区间 $[0, d]$ 分成若干相等的部分, 对时滞信息的获得有一定的保守性.

本文进一步讨论具有混合时滞的人工神经网络的指数稳定性问题. 通过将离散时滞区间 $[0, d]$ 分成 $[0, d_1]$ 和 $[d_1, d]$ 两段不等的区间(其中 $d_1 \in (0, d)$), 进一步运用积分不等式与改进的凸优化方法——倒数凸引理^[12]来处理 Lyapunov-Krasovskii 泛函中的积分项, 以期克服相关文献

时滞分解方法的保守性.

1 系统描述

考虑如下具有分布时滞的人工神经网络:

$$\dot{\mathbf{z}}(t) = -\mathbf{C}\mathbf{z}(t) + \mathbf{A}\mathbf{g}(\mathbf{z}(t)) + \mathbf{B}\mathbf{g}(\mathbf{z}(t - d(t))) + \mathbf{D} \int_{t-\tau(t)}^t \mathbf{g}(\mathbf{z}(s)) ds + \mathbf{u} \quad (1)$$

$$\mathbf{z}(t) = \Phi(t); t \in [-h, 0] \quad (2)$$

其中: $\mathbf{z}(t) = (z_1(t) \cdots z_n(t))^T \in \mathbf{R}^n$, 为神经元状态向量, $\mathbf{g}(\mathbf{z}(t)) = (g_1(z_1(t)) \cdots g_n(z_n(t)))^T \in \mathbf{R}^n$, 为神经元激活函数; $\mathbf{u} = (u_1 \cdots u_n)^T \in \mathbf{R}^n$, 是常值输入向量; $\mathbf{C} = \text{diag}\{c_1, \dots, c_n\} > \mathbf{0}$; $\mathbf{A} \in \mathbf{R}^{n \times n}$, 是连接权矩阵; $\mathbf{B}, \mathbf{D} \in \mathbf{R}^{n \times n}$ 是时滞连接权矩阵; $d(t), \tau(t)$ 为满足 $0 \leq d(t) \leq d, 0 \leq \tau(t) \leq \tau, \dot{d}(t) \leq u$ 的连续时变函数, 其中 d, τ, u 都是常数. 初始向量 $\Phi(t)$ 是有界的且在 $[-h, 0]$ 上连续可微, 其中 $h = \max\{d, \tau\}$. 神经元激活函数 $g_i(\cdot) (i = 1, 2, \dots, n)$ 满足

$$\rho_i^- \leq \frac{g_i(u) - g_i(v)}{u - v} \leq \rho_i^+;$$

$$\forall u, v \in \mathbf{R}, u \neq v, i = 1, \dots, n \quad (3)$$

其中 $\rho_i^-, \rho_i^+ (i = 1, \dots, n)$ 是常数.

根据 Brouwer 不动点定理, 神经网络(1)存

在平衡点, 假定 $\mathbf{z}^* = (z_1^* \cdots z_n^*)$ 是神经网络 (1) 的一个平衡点, 由文献[13] 知该平衡点是唯一平衡点. 通过坐标平移变换 $x(\cdot) = z(\cdot) - z^*(\cdot)$, 系统 (1) 转换为如下形式:

$$\dot{\mathbf{x}}(t) = -\mathbf{C}\mathbf{x}(t) + \mathbf{A}\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}\mathbf{f}(\mathbf{x}(t-d(t))) + \mathbf{D}\int_{t-\tau(t)}^t \mathbf{f}(\mathbf{x}(s))ds \quad (4)$$

其中 $\mathbf{x}(t) = (x_1(t) \cdots x_n(t))^T$, $\mathbf{f}(\mathbf{x}(t)) = (f_1(x_1(t)) \cdots f_n(x_n(t)))^T$, $f_i(x_i(t)) = g_i(x_i(t) + z_i^*) - g_i(z_i^*)$ ($i = 1, 2, \cdots, n$), 且 $f_i(0) = 0$ ($i = 1, \cdots, n$). 由不等式 (3), 可得

$$\rho_i^- \leq \frac{f_i(u) - f_i(v)}{u - v} \leq \rho_i^+; \quad \forall u, v \in \mathbf{R}, u \neq v, i = 1, \cdots, n \quad (5)$$

当 $v = 0$ 时, 有

$$\rho_i^- \leq \frac{f_i(u)}{u} \leq \rho_i^+; \quad \forall u \neq 0, i = 1, \cdots, n \quad (6)$$

定义 1 若变量 $k > 0, \beta > 0$ 且满足

$$\|\mathbf{x}(t)\| \leq \beta e^{-kt} \sup_{-h \leq s \leq 0} \|\mathbf{x}(s)\|; \quad \forall t > 0 \quad (7)$$

则系统 (4) 是全局指数稳定的.

引理 1^[14] 对于任意正定矩阵 $\mathbf{Z} \in \mathbf{R}^{n \times n}$ ($\mathbf{R}^{n \times n}$ 表示 $n \times n$ 的实矩阵), 标量 $h_2 > h_1 > 0$, 有以下不等式成立:

$$-(h_2 - h_1) \int_{t-h_2}^{t-h_1} \mathbf{x}^T(s) \mathbf{Z} \mathbf{x}(s) ds \leq - \int_{t-h_2}^{t-h_1} \mathbf{x}^T(s) ds \mathbf{Z} \int_{t-h_2}^{t-h_1} \mathbf{x}(s) ds \quad (8)$$

引理 2^[12] 设 $f_1, f_2, \cdots, f_N: \mathbf{R}^m \mapsto \mathbf{R}$ 是定义在 $\mathbf{R}^m$ ($\mathbf{R}^m$ 表示 m 维 Euclidean 空间) 的子集 $\mathbf{D}$ 上的正值, 则关于 f_i 的倒数凸组合, 如果有

$$g_{i,j}: \mathbf{R}^m \rightarrow \mathbf{R}, g_{j,i}(t) = g_{i,j}(t), \begin{pmatrix} f_i(t) & g_{i,j}(t) \\ g_{j,i}(t) & f_j(t) \end{pmatrix} \geq \mathbf{0}$$

则满足

$$\min_{\{a_i \mid a_i > 0, \sum_i a_i = 1\}} \sum_i \frac{1}{a_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i \neq j} g_{i,j}(t)$$

引理 3^[15] 以下不等式成立:

$$0 \leq \int_0^{x_i(t)} (f_i(s) - \rho_i^- s) ds \leq (f_i(x_i(t)) - \rho_i^- x_i(t)) x_i(t) \quad (9)$$

$$0 \leq \int_0^{x_i(t)} (\rho_i^+ s - f_i(s)) ds \leq (\rho_i^+ x_i(t) - f_i(x_i(t))) x_i(t) \quad (10)$$

2 指数稳定新判据

下面将给出一个基于时滞分段方法的具有更小保守性的指数稳定性新判据. 为了描述方便, 记 $\mathbf{e}_1^T = (\mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0})$, $\mathbf{e}_2^T = (\mathbf{0} \quad \mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0})$, $\cdots$, $\mathbf{e}_7^T = (\mathbf{0} \quad \cdots \quad \mathbf{0} \quad \mathbf{I})$, $\mathbf{e}_8 = -\mathbf{e}_1 \mathbf{C} + \mathbf{e}_5 \mathbf{A} + \mathbf{e}_6 \mathbf{B} + \mathbf{e}_7 \mathbf{D}$, $\xi^T(t) = \begin{pmatrix} \mathbf{x}^T(t) & \mathbf{x}^T(t-d(t)) & \mathbf{x}^T(t-d_1) & \mathbf{x}^T(t-d) & \mathbf{f}^T(\mathbf{x}(t)) & \mathbf{f}^T(\mathbf{x}(t-d(t))) \end{pmatrix} \int_{t-\tau}^t \mathbf{f}^T(\mathbf{x}(s)) ds$.

定理 1 对于给定的常数 d, d_1, τ, u 及指数衰减率 k , 对角矩阵 $\boldsymbol{\rho}_p = \text{diag}\{\rho_1^-, \cdots, \rho_n^-\}$, $\boldsymbol{\rho}_m = \text{diag}\{\rho_1^+, \cdots, \rho_n^+\}$, 系统 (4) 是全局指数稳定的, 如果存在对角矩阵 $\mathbf{K} = \text{diag}\{k_1, \cdots, k_n\}$, $\mathbf{L} = \text{diag}\{l_1, \cdots, l_n\}$, $\mathbf{H}_i = \text{diag}\{h_{i1}, \cdots, h_{in}\}$ ($i = 1, 2, 3$), n 维正定对称矩阵 $\mathbf{P}, \mathbf{Q}, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3, \mathbf{Q}_4, \mathbf{R}_1, \mathbf{R}_2, \mathbf{R}_3, \mathbf{Q}_{11}, \mathbf{Q}_{22}$, n 维的矩阵 $\mathbf{Q}_{12}, \mathbf{T}_1, \mathbf{T}_2$, 满足下面的线性矩阵不等式

$$\Xi_1 + \Xi_2 + \Xi_{3i} < \mathbf{0}; \quad i = 1, 2 \quad (11)$$

$$\Delta_i \geq \mathbf{0}; \quad i = 1, 2 \quad (12)$$

其中 $\mathbf{P} \geq (<) \mathbf{0}$ 表示 $\mathbf{P}$ 为半正定(负定) 矩阵,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix}, \Delta_1 = \begin{pmatrix} \mathbf{R}_1 & \mathbf{T}_1 \\ \mathbf{T}_1^T & \mathbf{R}_1 \end{pmatrix}, \Delta_2 = \begin{pmatrix} \mathbf{R}_2 & \mathbf{T}_2 \\ \mathbf{T}_2^T & \mathbf{R}_2 \end{pmatrix},$$

$$\begin{aligned} \Xi_1 = & 2ke_1 \mathbf{P} \mathbf{e}_1^T + 2e_1 \mathbf{P} \mathbf{e}_8^T + 4k((e_5 - e_1 \boldsymbol{\rho}_p) \mathbf{K} \mathbf{e}_1^T + (e_1 \boldsymbol{\rho}_m - e_5) \mathbf{L} \mathbf{e}_1^T) + 2(e_1 (\boldsymbol{\rho}_m \mathbf{L} - \boldsymbol{\rho}_p \mathbf{K}) + e_5 (\mathbf{K} - \mathbf{L})) \mathbf{e}_8^T + (e_1 \quad e_5) \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \times \\ & (e_1 \quad e_5)^T + e_1 \mathbf{Q}_1 \mathbf{e}_1^T + e_5 \mathbf{Q}_2 \mathbf{e}_5^T + e^{-2kd_1} e_3 \mathbf{Q}_3 \mathbf{e}_3^T + e_1 \mathbf{Q}_4 \mathbf{e}_1^T - e^{-2kd} e_4 \mathbf{Q}_3 \mathbf{e}_4^T - e^{-2kd_1} e_3 \mathbf{Q}_4 \mathbf{e}_3^T + d_1^2 e_8 \mathbf{R}_1 \mathbf{e}_8^T + (d - d_1) e_8 \mathbf{R}_2 \mathbf{e}_8^T + \tau^2 e_5 \mathbf{R}_3 \mathbf{e}_5^T - e^{-2k\tau} e_7 \mathbf{R}_3 \mathbf{e}_7^T \end{aligned}$$

$$\begin{aligned} \Xi_2 = & -2(e_5 - e_1 \boldsymbol{\rho}_p) \mathbf{H}_1 (e_5 - e_1 \boldsymbol{\rho}_m)^T - 2(e_6 - e_2 \boldsymbol{\rho}_p) \mathbf{H}_2 (e_6 - e_2 \boldsymbol{\rho}_m)^T - 2(e_5 - e_6 - (e_1 - e_2) \boldsymbol{\rho}_p) \mathbf{H}_3 \times \\ & (e_5 - e_6 - (e_1 - e_2) \boldsymbol{\rho}_m)^T \end{aligned}$$

$$\begin{aligned} \Xi_{31} = & -(1-u)e^{-2kd_1} (e_2 \quad e_6) \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \times \\ & (e_2 \quad e_6)^T - (1-u)(e^{-2kd_1} e_2 \mathbf{Q}_1 \mathbf{e}_2^T + e^{-2kd_1} e_6 \mathbf{Q}_2 \mathbf{e}_6^T) - e^{-2kd} (e_3 - e_4) \mathbf{R}_2 (e_3 - e_4)^T - e^{-2kd_1} (e_2 - e_3 \quad e_1 - e_2) \times \\ & \Delta_1 (e_2 - e_3 \quad e_1 - e_2)^T \end{aligned}$$

$$\Xi_{32} = -(1-u)e^{-2kd} (e_2 \quad e_6) \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \times$$

$$\begin{aligned} & (\mathbf{e}_2 \quad \mathbf{e}_6)^T - (1-u)(\mathbf{e}^{-2kd} \mathbf{e}_2 \mathbf{Q}_1 \mathbf{e}_2^T + \\ & \mathbf{e}^{-2kd} \mathbf{e}_6 \mathbf{Q}_2 \mathbf{e}_6^T) - \mathbf{e}^{-2kd_1} (\mathbf{e}_1 - \mathbf{e}_3) \mathbf{R}_1 (\mathbf{e}_1 - \mathbf{e}_3)^T - \\ & \mathbf{e}^{-2kd} (\mathbf{e}_2 - \mathbf{e}_4 \quad \mathbf{e}_3 - \mathbf{e}_2) \times \\ & \mathbf{A}_2 (\mathbf{e}_2 - \mathbf{e}_4 \quad \mathbf{e}_3 - \mathbf{e}_2)^T \end{aligned}$$

证明 构造如下 Lyapunov 泛函:

$$V(\mathbf{x}(t)) = V_1(\mathbf{x}(t)) + V_2(\mathbf{x}(t)) + V_3(\mathbf{x}(t)) + V_4(\mathbf{x}(t)) + V_5(\mathbf{x}(t))$$

其中

$$\begin{aligned} V_1(\mathbf{x}(t)) &= \mathbf{e}^{2kt} \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) + 2\mathbf{e}^{2kt} \times \\ & \sum_{i=1}^n \left(\int_0^{x_i(t)} k_i(f_i(s) - \rho_i^- s) + \right. \\ & \left. l_i(\rho_i^+ s - f_i(s)) \mathrm{d}s \right) \\ V_2(\mathbf{x}(t)) &= \int_{t-d(t)}^t \mathbf{e}^{2ks} \begin{pmatrix} \mathbf{x}(s) \\ \mathbf{f}(\mathbf{x}(s)) \end{pmatrix}^T \times \\ & \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \begin{pmatrix} \mathbf{x}(s) \\ \mathbf{f}(\mathbf{x}(s)) \end{pmatrix} \mathrm{d}s \end{aligned}$$

$$\begin{aligned} V_3(\mathbf{x}(t)) &= \int_{t-d(t)}^t [\mathbf{e}^{2ks} \mathbf{x}^T(s) \mathbf{Q}_1 \mathbf{x}(s) + \mathbf{e}^{2ks} \mathbf{f}^T(\mathbf{x}(s)) \times \\ & \mathbf{Q}_2 \mathbf{f}(\mathbf{x}(s))] \mathrm{d}s + \int_{t-d}^{t-d_1} [\mathbf{e}^{2ks} \mathbf{x}(s)^T \times \\ & \mathbf{Q}_3 \mathbf{x}(s)] \mathrm{d}s + \int_{t-d_1}^t \mathbf{e}^{2ks} \mathbf{x}^T(s) \mathbf{Q}_4 \mathbf{x}(s) \mathrm{d}s \end{aligned}$$

$$\begin{aligned} V_4(\mathbf{x}(t)) &= d_1 \int_{-d_1}^t \int_{t+\theta}^t \mathbf{e}^{2ks} \dot{\mathbf{x}}^T(s) \mathbf{R}_1 \dot{\mathbf{x}}(s) \mathrm{d}s \mathrm{d}\theta + \\ & (d-d_1) \int_{-d}^{t-d_1} \int_{t+\theta}^t \mathbf{e}^{2ks} \dot{\mathbf{x}}^T(s) \mathbf{R}_2 \dot{\mathbf{x}}(s) \mathrm{d}s \mathrm{d}\theta \end{aligned}$$

$$V_5(\mathbf{x}(t)) = \tau \int_{-\tau}^0 \int_{t+\theta}^t \mathbf{e}^{2ks} \mathbf{f}^T(\mathbf{x}(s)) \mathbf{R}_3 \mathbf{f}(\mathbf{x}(s)) \mathrm{d}s \mathrm{d}\theta$$

计算 $V_i(\mathbf{x}(t)) (i=1, \dots, 5)$ 的导数, 有

$$\begin{aligned} \dot{V}_1(\mathbf{x}(t)) &= 2k\mathbf{e}^{2kt} \mathbf{x}^T(t) \mathbf{P} \mathbf{x}(t) + 2\mathbf{e}^{2kt} \mathbf{x}^T(t) \mathbf{P} \dot{\mathbf{x}}(t) + \\ & 2\mathbf{e}^{2kt} ((\mathbf{f}^T(\mathbf{x}(t)) - \mathbf{x}^T(t) \mathbf{p}_p) \mathbf{K} \dot{\mathbf{x}}(t) + \\ & (\mathbf{x}^T(t) \mathbf{p}_m - \mathbf{f}^T(\mathbf{x}(t))) \mathbf{L} \dot{\mathbf{x}}(t)) + \\ & 4k\mathbf{e}^{2kt} \sum_{i=1}^n \left(\int_0^{x_i(t)} k_i(f_i(s) - \rho_i^- s) + \right. \\ & \left. l_i(\rho_i^+ s - f_i(s)) \mathrm{d}s \right) \end{aligned}$$

$$\begin{aligned} \dot{V}_2(\mathbf{x}(t)) &= \mathbf{e}^{2kt} \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{f}(\mathbf{x}(t)) \end{pmatrix}^T \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \times \\ & \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{f}(\mathbf{x}(t)) \end{pmatrix} - (1-\dot{d}(t)) \mathbf{e}^{2k(t-d(t))} \times \\ & \begin{pmatrix} \mathbf{x}(t-d(t)) \\ \mathbf{f}(\mathbf{x}(t-d(t))) \end{pmatrix}^T \begin{pmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{12}^T & \mathbf{Q}_{22} \end{pmatrix} \times \\ & \begin{pmatrix} \mathbf{x}(t-d(t)) \\ \mathbf{f}(\mathbf{x}(t-d(t))) \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \dot{V}_3(\mathbf{x}(t)) &= \mathbf{e}^{2kt} \mathbf{x}^T(t) \mathbf{Q}_1 \mathbf{x}(t) + \mathbf{e}^{2kt} \mathbf{f}^T(\mathbf{x}(t)) \times \\ & \mathbf{Q}_2 \mathbf{f}(\mathbf{x}(t)) - (1-\dot{d}(t)) (\mathbf{e}^{2k(t-d(t))} \times \\ & \mathbf{x}^T(t-d(t)) \mathbf{Q}_1 \mathbf{x}(t-d(t)) + \\ & \mathbf{e}^{2k(t-d(t))} \mathbf{f}^T(\mathbf{x}(t-d(t))) \mathbf{Q}_2 \mathbf{f}(\mathbf{x}(t-d(t)))) + \mathbf{e}^{2k(t-d_1)} \mathbf{x}^T(t-d_1) \mathbf{Q}_3 \mathbf{x}(t-d_1) - \mathbf{e}^{2k(t-d)} \mathbf{x}^T(t-d) \mathbf{Q}_3 \mathbf{x}(t-d) + \\ & \mathbf{e}^{2kt} \mathbf{x}^T(t) \mathbf{Q}_4 \mathbf{x}(t) - \mathbf{e}^{2k(t-d_1)} \mathbf{x}^T(t-d_1) \times \\ & \mathbf{Q}_4 \mathbf{x}(t-d_1) \end{aligned}$$

$$\begin{aligned} \dot{V}_4(\mathbf{x}(t)) &= d_1^2 \mathbf{e}^{2kt} \dot{\mathbf{x}}^T(t) \mathbf{R}_1 \dot{\mathbf{x}}(t) + (d-d_1)^2 \mathbf{e}^{2kt} \times \\ & \dot{\mathbf{x}}^T(t) \mathbf{R}_2 \dot{\mathbf{x}}(t) - d_1 \int_{t-d_1}^t (\mathbf{e}^{2ks} \dot{\mathbf{x}}^T(s) \mathbf{R}_1 \times \\ & \dot{\mathbf{x}}(s)) \mathrm{d}s - (d-d_1) \int_{t-d}^{t-d_1} (\mathbf{e}^{2ks} \dot{\mathbf{x}}^T(s) \mathbf{R}_2 \times \\ & \dot{\mathbf{x}}(s)) \mathrm{d}s \end{aligned}$$

$$\begin{aligned} \dot{V}_5(\mathbf{x}(t)) &= \tau^2 \mathbf{e}^{2kt} \mathbf{f}^T(\mathbf{x}(t)) \mathbf{R}_3 \mathbf{f}(\mathbf{x}(t)) - \\ & \tau \int_{t-\tau}^t \mathbf{e}^{2ks} \mathbf{f}^T(\mathbf{x}(s)) \mathbf{R}_3 \mathbf{f}(\mathbf{x}(s)) \mathrm{d}s \end{aligned}$$

另外, 由式(5)、(6)知, 对任意的正定对角矩阵 $\mathbf{H}_i = \text{diag}\{h_{i1}, \dots, h_{in}\} (i=1, 2, 3)$, 有

$$\begin{aligned} 0 &\leq -2 \sum_{i=1}^n h_{1i} [f_i(x_i(t)) - \rho_i^- x_i(t)] [f_i(x_i(t)) - \\ & \rho_i^+ x_i(t)] - 2 \sum_{i=1}^n \{h_{2i} [f_i(x_i(t-\tau(t))) - \\ & \rho_i^- x_i(t-\tau(t))] [f_i(x_i(t-\tau(t))) - \rho_i^+ x_i(t-\tau(t))]\} - 2 \sum_{i=1}^n \{h_{3i} [f_i(x_i(t)) - f_i(x_i(t-\tau(t))) - \rho_i^- (x_i(t) - x_i(t-\tau(t)))] \times \\ & [f_i(x_i(t)) - f_i(x_i(t-\tau(t))) - \rho_i^+ (x_i(t) - x_i(t-\tau(t)))]\} = \\ & -2(\mathbf{f}(\mathbf{x}(t)) - \mathbf{p}_m \mathbf{x}(t))^T \mathbf{H}_1 (\mathbf{f}(\mathbf{x}(t)) - \mathbf{p}_p \mathbf{x}(t)) - 2(\mathbf{f}(\mathbf{x}(t-\tau(t))) - \mathbf{p}_m \mathbf{x}(t-\tau(t)))^T \mathbf{H}_2 (\mathbf{f}(\mathbf{x}(t-\tau(t))) - \mathbf{p}_p \mathbf{x}(t-\tau(t))) - 2(\mathbf{f}(\mathbf{x}(t)) - \mathbf{f}(\mathbf{x}(t-\tau(t))) - \mathbf{p}_m (\mathbf{x}(t) - \mathbf{x}(t-\tau(t))))^T \mathbf{H}_3 (\mathbf{f}(\mathbf{x}(t)) - \mathbf{f}(\mathbf{x}(t-\tau(t))) - \mathbf{p}_p (\mathbf{x}(t) - \mathbf{x}(t-\tau(t)))) \end{aligned}$$

注意到 $\dot{d}(t) \leq u$, 利用式(8)~(10)以及引理2处理上述导数项中的积分项, 将 $\dot{\mathbf{x}}(t) = -\mathbf{C}\mathbf{x}(t) + \mathbf{A}\mathbf{f}(\mathbf{x}(t)) + \mathbf{B}\mathbf{f}(\mathbf{x}(t-\tau(t))) + \mathbf{D} \int_{t-\tau(t)}^t \mathbf{f}(\mathbf{x}(s)) \mathrm{d}s$

代入所求得导数中, 得

当时滞 $d(t) \in [0, d_1]$ 时

$$\dot{V}(\mathbf{x}(t)) \leq \mathbf{e}^{2kt} \boldsymbol{\xi}^T(t) (\boldsymbol{\Xi}_1 + \boldsymbol{\Xi}_2 + \boldsymbol{\Xi}_{31}) \boldsymbol{\xi}(t)$$

当时滞 $d(t) \in [d_1, d]$ 时

$\dot{V}(\boldsymbol{x}(t)) \leqslant \mathrm{e}^{2kt} \boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{\Xi}_1 + \boldsymbol{\Xi}_2 + \boldsymbol{\Xi}_{32}) \boldsymbol{\xi}(t)$
因此, 当线性矩阵不等式 (11)、(12) 成立时,
 $\dot{V}(\boldsymbol{x}(t)) < 0$, 则系统 (4) 是全局渐近稳定的.

由 $\dot{V}(\boldsymbol{x}(t)) < 0$ 可得, $V(\boldsymbol{x}(t)) \leqslant V(\boldsymbol{x}(0))$.
其中

$$\begin{aligned} V_1(\boldsymbol{x}(0)) &\leqslant \lambda_{\max}(\boldsymbol{P}) \|\boldsymbol{x}(0)\|^2 + 2(\boldsymbol{f}(\boldsymbol{x}(0)) - \\ &\quad \boldsymbol{\rho}_{\mathrm{p}} \boldsymbol{x}(0))^{\mathrm{T}} \boldsymbol{K} \boldsymbol{x}(0) + 2(\boldsymbol{\rho}_{\mathrm{m}} \boldsymbol{x}(0) - \\ &\quad \boldsymbol{f}(\boldsymbol{x}(0)))^{\mathrm{T}} \boldsymbol{L} \boldsymbol{x}(0) \leqslant \\ &\quad \lambda_{\max}(\boldsymbol{P}) \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 + 2\lambda_{\max}(\boldsymbol{\rho}_{\mathrm{m}} - \\ &\quad \boldsymbol{\rho}_{\mathrm{p}})(\lambda_{\max}(\boldsymbol{K}) + \lambda_{\max}(\boldsymbol{L})) \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

$$\begin{aligned} V_2(\boldsymbol{x}(0)) &\leqslant \int_{-d}^0 \mathrm{e}^{2ks} \begin{pmatrix} \boldsymbol{x}(s) \\ \boldsymbol{f}(\boldsymbol{x}(s)) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{12}^{\mathrm{T}} & \boldsymbol{Q}_{22} \end{pmatrix} \times \\ &\quad \begin{pmatrix} \boldsymbol{x}(s) \\ \boldsymbol{f}(\boldsymbol{x}(s)) \end{pmatrix} \mathrm{d}s \leqslant \\ &\quad d(1 + \rho)^2 \lambda_{\max} \left(\begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{12}^{\mathrm{T}} & \boldsymbol{Q}_{22} \end{pmatrix} \right) \times \\ &\quad \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

其中 $\rho = \max_{1 \leqslant i \leqslant n} \{|\rho_i^-|, |\rho_i^+|\}$.

$$\begin{aligned} V_3(\boldsymbol{x}(0)) &\leqslant (d\lambda_{\max}(\boldsymbol{Q}_1) + d\rho^2 \lambda_{\max}(\boldsymbol{Q}_2) + (d - \\ &\quad d_1)\lambda_{\max}(\boldsymbol{Q}_3) + d_1\lambda_{\max}(\boldsymbol{Q}_4)) \times \\ &\quad \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

$$\begin{aligned} V_4(\boldsymbol{x}(0)) &\leqslant \lambda_{\max}(\boldsymbol{R}_1) d_1 \int_{-d_1}^0 \int_{\theta}^0 \dot{\boldsymbol{x}}^{\mathrm{T}}(s) \dot{\boldsymbol{x}}(s) \mathrm{d}s \mathrm{d}\theta + \\ &\quad \lambda_{\max}(\boldsymbol{R}_2) (d - d_1) \int_{-d}^{-d_1} \int_{\theta}^0 (\dot{\boldsymbol{x}}^{\mathrm{T}}(s) \times \\ &\quad \dot{\boldsymbol{x}}(s)) \mathrm{d}s \mathrm{d}\theta \end{aligned}$$

$$\begin{aligned} \dot{\boldsymbol{x}}^{\mathrm{T}}(s) \dot{\boldsymbol{x}}(s) &\leqslant 4(\lambda_{\max}(\boldsymbol{C}^{\mathrm{T}} \boldsymbol{C}) + \rho^2 \lambda_{\max}(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A}) + \\ &\quad \rho^2 \lambda_{\max}(\boldsymbol{B}^{\mathrm{T}} \boldsymbol{B}) + \tau^2 \rho^2 \lambda_{\max}(\boldsymbol{D}^{\mathrm{T}} \boldsymbol{D})) \times \\ &\quad \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

$$\begin{aligned} V_5(\boldsymbol{x}(0)) &\leqslant \tau \lambda_{\max}(\boldsymbol{R}_3) \int_{-\tau}^0 \int_{\theta}^0 \boldsymbol{f}^{\mathrm{T}}(\boldsymbol{x}(s)) \boldsymbol{f}(\boldsymbol{x}(s)) \times \\ &\quad \mathrm{d}s \mathrm{d}\theta \leqslant \frac{\rho^2 \tau^3}{2} \lambda_{\max}(\boldsymbol{R}_3) \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

$$\begin{aligned} V(\boldsymbol{x}(0)) &= V_1(\boldsymbol{x}(0)) + V_2(\boldsymbol{x}(0)) + V_3(\boldsymbol{x}(0)) + \\ &\quad V_4(\boldsymbol{x}(0)) + V_5(\boldsymbol{x}(0)) \leqslant \\ &\quad \alpha \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|^2 \end{aligned}$$

$$\begin{aligned} \alpha &= \lambda_{\max}(\boldsymbol{P}) + 2\lambda_{\max}(\boldsymbol{\rho}_{\mathrm{m}} - \boldsymbol{\rho}_{\mathrm{p}})(\lambda_{\max}(\boldsymbol{K}) + \lambda_{\max}(\boldsymbol{L})) + \\ &\quad d(1 + \rho)^2 \lambda_{\max} \left(\begin{pmatrix} \boldsymbol{Q}_{11} & \boldsymbol{Q}_{12} \\ \boldsymbol{Q}_{12}^{\mathrm{T}} & \boldsymbol{Q}_{22} \end{pmatrix} \right) + [d\lambda_{\max}(\boldsymbol{Q}_1) + \\ &\quad d\rho^2 \lambda_{\max}(\boldsymbol{Q}_2) + (d - d_1)\lambda_{\max}(\boldsymbol{Q}_3) + d_1\lambda_{\max}(\boldsymbol{Q}_4)] + \\ &\quad 4[\lambda_{\max}(\boldsymbol{R}_1) d_1 + \lambda_{\max}(\boldsymbol{R}_2) (d - d_1)] [\lambda_{\max}(\boldsymbol{C}^{\mathrm{T}} \boldsymbol{C}) + \end{aligned}$$

$$\begin{aligned} &\rho^2 \lambda_{\max}(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{A}) + \rho^2 \lambda_{\max}(\boldsymbol{B}^{\mathrm{T}} \boldsymbol{B}) + \\ &\quad \tau^2 \rho^2 \lambda_{\max}(\boldsymbol{D}^{\mathrm{T}} \boldsymbol{D})] + \frac{\rho^2 \tau^3}{2} \lambda_{\max}(\boldsymbol{R}_3) \end{aligned}$$

又 $V(\boldsymbol{x}(t)) \geqslant \mathrm{e}^{2kt} \lambda_{\min}(\boldsymbol{P}) \|\boldsymbol{x}(t)\|^2$, 故 $\|\boldsymbol{x}(t)\| \leqslant$
 $\sqrt{\frac{\alpha}{\lambda_{\min}(\boldsymbol{P})}} \mathrm{e}^{-kt} \sup_{-h \leqslant s \leqslant 0} \|\boldsymbol{x}(s)\|$.

由定义 1 可知, 系统 (4) 是指数稳定的, 定理成立. 证毕.

3 数值实例

下面给出两个数值实例说明定理所给条件的优越性.

例 1 考虑系统 (4), 其中

$$\begin{aligned} \boldsymbol{C} &= \begin{pmatrix} 2.3 & 0 & 0 \\ 0 & 3.4 & 0 \\ 0 & 0 & 2.5 \end{pmatrix}, \\ \boldsymbol{A} &= \begin{pmatrix} 0.9 & -1.5 & 0.1 \\ -1.2 & 1.0 & 0.2 \\ 0.2 & 0.3 & 0.8 \end{pmatrix}, \\ \boldsymbol{B} &= \begin{pmatrix} 0.8 & 0.6 & 0.2 \\ 0.5 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.5 \end{pmatrix}, \boldsymbol{D} = \begin{pmatrix} 0.3 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{pmatrix}, \\ \boldsymbol{\rho}_{\mathrm{p}} &= \text{diag}\{0, 0, 0\}, \boldsymbol{\rho}_{\mathrm{m}} = \text{diag}\{0.2, 0.2, 0.2\} \end{aligned}$$

表 1 给出了当指数衰减率 $k = 0, u$ 未知, $d = \tau$ 时, 利用定理所得时滞上界与文献 [11, 15-17] 的比较, 其中文献 [15-17] 不划分区间, 文献 [11] 均分区间, 本文以 $d_1 = 5.21$ 划分区间. 由表 1 可以看出, 定理所得结果具有更小的保守性. 为了验证所得结果, 取 $\boldsymbol{f}(\boldsymbol{x}(t)) =$
 $\begin{pmatrix} 0.2 \tanh(x_1(t)) \\ 0.2 \tanh(x_2(t)) \\ 0.2 \tanh(x_3(t)) \end{pmatrix}, d(t) = 1.5 \sin^2 t, \tau(t) =$
 $1.5 \cos^2 t$, 图 1 为例 1 中系统的仿真曲线图. 由图 1 可以看出, 当时间逐渐增加时, 状态响应曲线趋向于 0.

表 1 系统稳定的最大时滞上界 d

Tab. 1 Allowable delay upper bound d

方法	时滞上界 d
文献[16]	1.833
文献[17]	5.068
文献[15]	6.938
文献[11]	9.338
本文	10.440

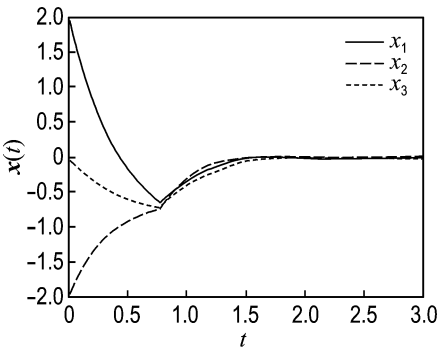


图 1 例 1 中 $x_1(t)$ 、 $x_2(t)$ 、 $x_3(t)$ 的状态响应曲线

Fig.1 State responses curves of $x_1(t)$, $x_2(t)$ and $x_3(t)$ in Example 1

例 2 考虑系统(4),其中

$$C = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 7 \end{pmatrix}, A = \begin{pmatrix} 1.2 & -0.8 & 0.6 \\ 0.5 & -1.5 & 0.7 \\ -0.8 & -1.2 & -1.4 \end{pmatrix},$$
$$B = \begin{pmatrix} -1.4 & 0.9 & 0.5 \\ -0.6 & 1.2 & 0.8 \\ 0.5 & -0.7 & 1.1 \end{pmatrix},$$
$$D = \begin{pmatrix} 1.8 & 0.7 & -0.8 \\ 0.6 & 1.4 & 1.0 \\ -0.4 & -0.6 & 1.2 \end{pmatrix},$$

$\rho_p = \text{diag}\{-1.2, 0, -2.4\}$, $\rho_m = \text{diag}\{0, 1.4, 0\}$

当参数 τ, u, d 为变量时,表 2 给出了由定理所得的最大指数衰减率与文献[11,15,17] 所得结果的比较.由表 2 可以看出,本文所得结果与上述文献相比有了很大的提高,因此使得指数稳定性结果具有更小的保守性.为了验证所得结果,取

$$f(x(t)) = \begin{pmatrix} -1.2 \tanh(x_1(t)) \\ 1.4 \tanh(x_2(t)) \\ -2.4 \tanh(x_3(t)) \end{pmatrix}, \quad d(t) =$$

$0.25 \sin^2 t, \tau(t) = 0.2 \cos^2 t$, 图 2 即为例 2 中系统

表 2 在不同的 τ, u, d 下,系统稳定的最大指数衰减率 k

Tab.2 Allowable exponential convergence rate index k with different τ, u and d

d	τ	u	k			
			文献[17]	文献[15]	文献[11]	本文
0.5	0.2	0	0.46	0.58	0.67	0.74($d_1=0.25$)
0.5	0.2	0.5	0.21	0.35	0.45	0.53($d_1=0.17$)
0.6	0.2	0.5	0.06	0.20	0.29	0.33($d_1=0.21$)
0.8	0.2	0.5	0	0.05	0.11	0.13($d_1=0.21$)

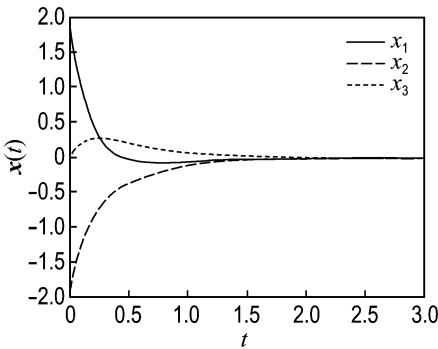


图 2 例 2 中 $x_1(t)$ 、 $x_2(t)$ 、 $x_3(t)$ 的状态响应曲线

Fig.2 State responses curves of $x_1(t)$, $x_2(t)$ and $x_3(t)$ in Example 2

的状态响应曲线.由图 2 可以看出,随着时间的增加状态响应曲线也逐渐趋向于 0.

4 结 语

本文讨论了具有分布时滞的人工神经网络的指数稳定性问题.通过对时滞区间进行不等分割并运用倒数凸引理得到了基于线性矩阵不等式的全局指数稳定性新判据.数值实例说明了所得结果的有效性与更小的保守性.在后续的研究中,将讨论将时滞区间分为不等的 3 个子区间时,对神经网络稳定性的影响.

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Exponential stability analysis for a class of neural networks with mixed delays

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Abstract: The problem of exponential stability analysis for artificial neural networks with mixed delays (including discrete and distributed time-varying delays) is discussed. By dividing the delay interval into two unequal subintervals and using the reciprocal convex approach, an improved delay-dependent exponential-stability criterion is derived in terms of the linear matrix inequalities (LMIs). Finally, numerical examples illustrate the effectiveness and less conservatism of the obtained conditions.

Key words: mixed delay; exponential stability; reciprocal convex approach; linear matrix inequality (LMI)