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含参集值向量均衡问题近似解映射的 Lipschitz 连续性

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摘要： 在赋范线性空间中研究了含参集值向量均衡问题.在引入含参集值向量均衡问题近似有效解的基础上，讨论了含参集值向量均衡问题近似解映射的 Lipschitz 连续性.借助标量化方法，得到了含参集值向量均衡问题近似解映射的 Lipschitz 连续的充分性定理.作为应用，研究了含参集值向量优化问题近似解映射的 Lipschitz 连续性，给出了含参集值向量优化问题近似解映射的 Lipschitz 连续的充分性条件.

关键词： Lipschitz 连续性；集值向量均衡问题；近似有效解；向量优化问题

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0 引 言

向量均衡问题解映射的稳定性分析是优化理论和应用中的一个重要课题.在一般情况下，(含参)向量均衡问题的解映射具有一定的定性或定量属性，如上半连续性、下半连续性、连续性、Hölder 连续性和 Lipschitz 连续性.对(含参)向量均衡问题解映射 Lipschitz 连续性的研究，不仅可以发展向量优化问题的理论与算法，而且可以为向量优化在交通均衡、工程管理、工程技术、资源分配等领域的应用提供重要的理论依据.

近年来，学者们密切关注对(含参)向量均衡问题解映射 Hölder 连续性和 Lipschitz 连续性的研究[1-16]. Li 等[1]研究了含参均衡问题近似解映射的 Lipschitz 连续性. Anh 等[2]建立了含参均衡问题近似有效解映射的 Hölder 连续性定理. Sadeqi 等[3]讨论了含参广义向量均衡问题近似有效解映射的 Lipschitz 连续性. Peng 等[4]运用标量化方法得到了含参广义 Ky Fan 不等式弱有效近似解映射 Hölder 连续性充分条件.最近，在文献[5]中，Anh 等在赋范空间中给出了含参向量均衡问题近似解映射的 Hölder 连续性定理.值得关注的是文献[6-7]通过解映射的信息讨论了含参向量均衡问题解映射的 Hölder 连续性，文献[7-12]借助映射的各种单调性得到了含参向量均衡问题解映射的 Hölder 连续性定理，文献[13-15]运用非线性标量化函数研究了含参向量均衡问题弱有效解映射的 Hölder 连续性.在文献[16]中，Han 在赋范线性空间中于不具单调性的适当条件下，讨论了含参广义向量均衡问题弱近似有效解映射和强近似有效解映射的 Lipschitz 连续性.受文献[1,3,5,16]思想的启发，本文在赋范线性空间中研究含参集值向量均衡问题近似解映射的 Lipschitz 连续性及应用.

1 准备工作

设 $\boldsymbol{X}$、$\boldsymbol{Y}$、$\boldsymbol{Z}$、$\boldsymbol{W}$ 为赋范线性空间，$\|\cdot\|$ 和 $d(\cdot,\cdot)$ 分别表示空间中的范数和距离，$\boldsymbol{C}\subset\boldsymbol{Y}$ 为闭凸点锥且 $\boldsymbol{C}$ 的拓扑内部 $\mathrm{int}(\boldsymbol{C})\neq\varnothing$.设 $\boldsymbol{Y}^*$ 为 $\boldsymbol{Y}$ 拓扑对偶空间，$\boldsymbol{C}^*$ 为 $\boldsymbol{C}$ 的拓扑对偶锥且定义为 $\boldsymbol{C}^*:=\{\boldsymbol{f}\in\boldsymbol{Y}^*:\boldsymbol{f}(\boldsymbol{c})\geqslant\boldsymbol{0},\forall\,\boldsymbol{c}\in\boldsymbol{C}\}$，$\boldsymbol{B}_0\subset\boldsymbol{Y}$ 为 $\boldsymbol{Y}$ 中以零点为中心的闭单位球，以及 $R_+=\{t\in\mathbf{R}:t\geqslant 0\}$.

设 $\boldsymbol{E}\subset\boldsymbol{Y}$ 为非空子集，$\boldsymbol{E}$ 的直径 $\mathrm{diam}(\boldsymbol{E})=\sup\limits_{\boldsymbol{x},\boldsymbol{z}\in\boldsymbol{E}}\|\boldsymbol{x}-\boldsymbol{z}\|$，$\boldsymbol{E}$ 的闭包和锥包分别记为 $\mathrm{cl}(\boldsymbol{E})$ 和

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$\mathrm{cone}(\boldsymbol{E})$，且 $\mathrm{cone}(\boldsymbol{E})=\{t\boldsymbol{\alpha}:\boldsymbol{\alpha}\in\boldsymbol{E},t\in R_+\}$.

$\boldsymbol{C}$ 的非空凸子集 $\boldsymbol{B}$ 称为 $\boldsymbol{C}$ 的基，若 $\boldsymbol{C}=\mathrm{cone}(\boldsymbol{B})$ 且 $0\notin\mathrm{cl}(\boldsymbol{B})$. $\boldsymbol{C}^*$ 的拟内部为 $\boldsymbol{C}^{\#}:=\{\boldsymbol{f}\in\boldsymbol{Y}^*:\boldsymbol{f}(\boldsymbol{c})>\boldsymbol{0},\forall\boldsymbol{c}\in\boldsymbol{C}\backslash\boldsymbol{0}\}$，易知，$\boldsymbol{C}$ 有基当且仅当 $\boldsymbol{C}^{\#}\neq\varnothing$. 设 $\boldsymbol{e}\in\mathrm{int}(\boldsymbol{C})$，则 $\boldsymbol{B}_e^*:=\{\boldsymbol{f}\in\boldsymbol{C}^*:\boldsymbol{f}(\boldsymbol{e})=\boldsymbol{1}\}$ 为 $\boldsymbol{C}^*$ 的弱紧基.

设 $\boldsymbol{F}:\boldsymbol{D}\times\boldsymbol{D}\times\boldsymbol{M}\subset\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{Z}\to2^{\boldsymbol{Y}}\backslash\{\varnothing\}$ 为集值映射，$\boldsymbol{K}:\boldsymbol{\Lambda}\subset\boldsymbol{W}\to2^{\boldsymbol{D}}\backslash\{\varnothing\}$ 为集值映射，对每个 $(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{\Lambda}\times\boldsymbol{M}$，讨论含参集值向量均衡问题(简记为(PSVEP))：找 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\cap(-\boldsymbol{\Omega})=\varnothing,\ \forall\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$$

其中 $\boldsymbol{\Omega}\cup\boldsymbol{0}$ 为 $\boldsymbol{Y}$ 中的凸锥.

设 $\boldsymbol{e}\in\mathrm{int}(\boldsymbol{C})$，对每个 $(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\in R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$，记(PSVEP)的近似弱有效解的全体为 $\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$，即

$$\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{\lambda}):(\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu})+\varepsilon\cdot\boldsymbol{e})\cap(-\mathrm{int}(\boldsymbol{C}))=\varnothing,\forall\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})\}$$

对每个 $\boldsymbol{f}\in\boldsymbol{B}_e^*$ 及 $(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\in R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$，记(PSVEP)的 $\boldsymbol{f}$-近似解的全体为 $\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$，即

$$\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{\lambda}):\boldsymbol{f}(\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu}))+\varepsilon\subset R_+,\forall\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})\}$$

特别地

(1)对任何 $\boldsymbol{\lambda}\in\boldsymbol{M}$，设 $\boldsymbol{K}(\boldsymbol{\lambda})=\boldsymbol{K},\boldsymbol{Y}=\boldsymbol{R},\boldsymbol{C}=[0,+\infty)$，$\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{Z}\to\boldsymbol{R}$，则(PSVEP)就是文献[1]中讨论的标量均衡问题：找 $\boldsymbol{x}_0\in\boldsymbol{K}$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\geqslant\boldsymbol{0},\ \forall\boldsymbol{y}\in\boldsymbol{K}$$

(2)对任何 $\boldsymbol{\lambda}\in\boldsymbol{M}$，设 $\boldsymbol{Y}=\boldsymbol{R},\boldsymbol{C}=[0,+\infty)$，$\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{Z}\to\boldsymbol{R}$ 时，则(PSVEP)就是文献[2]中研究的含参均衡问题：找 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\geqslant\boldsymbol{0},\ \forall\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$$

(3)当 $\boldsymbol{K}=\boldsymbol{D}$ 为 $\boldsymbol{X}$ 的非空凸子集时，则(PSVEP)就是文献[3]中讨论的弱向量均衡问题：找 $\boldsymbol{x}_0\in\boldsymbol{D}$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y})\cap(-\mathrm{int}(\boldsymbol{C}))=\varnothing,\ \forall\boldsymbol{y}\in\boldsymbol{D}$$

其中 $\boldsymbol{F}:\boldsymbol{D}\times\boldsymbol{D}\to2^{\boldsymbol{Y}}\backslash\{\varnothing\}$ 为集值映射.

定义 1[16]　(1)设 $L>0$，$\boldsymbol{F}:\boldsymbol{D}\times\boldsymbol{D}\times\boldsymbol{M}\subset\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{Z}\to2^{\boldsymbol{Y}}\backslash\{\varnothing\}$ 为集值映射，称 $\boldsymbol{F}$ 在 $\boldsymbol{\mu}_0\in\boldsymbol{M}$ 处关于 $\boldsymbol{D}\times\boldsymbol{D}$ 为 L-Lipschitz 一致连续当且仅当存在 $\boldsymbol{\mu}_0$ 的凸邻域 $\boldsymbol{V}(\boldsymbol{\mu}_0)\subset\boldsymbol{M}$，使得对任何 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{V}(\boldsymbol{\mu}_0)$ 及 $(\boldsymbol{x},\boldsymbol{y})\in\boldsymbol{D}\times\boldsymbol{D}$，有

$$\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu}_1)\subset\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu}_2)+L\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\boldsymbol{B}_0$$

(2)设 $\boldsymbol{K}:\boldsymbol{\Lambda}\subset\boldsymbol{W}\to2^{\boldsymbol{D}}\backslash\{\varnothing\}$ 为集值映射，称 $\boldsymbol{K}$ 在 $\boldsymbol{\lambda}_0\in\boldsymbol{\Lambda}$ 处为 L-Lipschitz 连续当且仅当存在 $\boldsymbol{\lambda}_0$ 凸邻域 $\boldsymbol{U}(\boldsymbol{\lambda}_0)\subset\boldsymbol{\Lambda}$，使得对任何的 $\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$，有

$$\boldsymbol{K}(\boldsymbol{\lambda}_1)\subset\boldsymbol{K}(\boldsymbol{\lambda}_2)+L\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\boldsymbol{B}_0$$

定义 2[17]　设 $(\boldsymbol{X},\boldsymbol{d})$ 为度量空间，$\boldsymbol{A},\boldsymbol{B}\subset\boldsymbol{X}$ 为非空子集，$\boldsymbol{A}$ 与 $\boldsymbol{B}$ 的 Hausdorff 距离定义为

$$H(\boldsymbol{A},\boldsymbol{B}):=\max(H^*(\boldsymbol{A},\boldsymbol{B}),H^*(\boldsymbol{B},\boldsymbol{A}))$$

其中 $H^*(\boldsymbol{A},\boldsymbol{B}):=\sup\limits_{\boldsymbol{a}\in\boldsymbol{A}}d(\boldsymbol{a},\boldsymbol{B})$，满足 $d(\boldsymbol{a},\boldsymbol{B}):=\inf\limits_{\boldsymbol{b}\in\boldsymbol{B}}d(\boldsymbol{a},\boldsymbol{b})$.

定义 3[18]　设 $\boldsymbol{D}\subset\boldsymbol{X}$ 为非空凸子集，$\boldsymbol{M}\subset\boldsymbol{Z}$ 为非空子集，$\boldsymbol{F}:\boldsymbol{D}\times\boldsymbol{D}\times\boldsymbol{M}\subset\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{Z}\to2^{\boldsymbol{Y}}\backslash\{\varnothing\}$ 为集值映射，对每个 $\boldsymbol{\mu}\in\boldsymbol{M}$，

(1)$\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$ 在 $\boldsymbol{D}$ 上 $\boldsymbol{C}$-凸当且仅当对任何的 $\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{D}$ 及 $t\in[0,1]$，有

$$t\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{\mu})+(1-t)\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_2,\boldsymbol{\mu})\subset\boldsymbol{F}(\boldsymbol{x},t\boldsymbol{x}_1+(1-t)\boldsymbol{x}_2,\boldsymbol{\mu})+\boldsymbol{C}$$

(2)$\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$ 在 $\boldsymbol{D}$ 上 $\boldsymbol{C}$-凹当且仅当对任何的 $\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{D}$ 及 $t\in[0,1]$，有

$$\boldsymbol{F}(\boldsymbol{x},t\boldsymbol{x}_1+(1-t)\boldsymbol{x}_2,\boldsymbol{\mu})\subset t\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{\mu})+(1-t)\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_2,\boldsymbol{\mu})+\boldsymbol{C}$$

(3)$\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$ 在 $\boldsymbol{D}$ 上 $\boldsymbol{C}$-类凸当且仅当对任何的 $\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{D}$ 及 $t\in[0,1]$，存在 $\boldsymbol{x}_3\in\boldsymbol{D}$，使得

$$t\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_1,\boldsymbol{\mu})+(1-t)\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_2,\boldsymbol{\mu})\subset\boldsymbol{F}(\boldsymbol{x},\boldsymbol{x}_3,\boldsymbol{\mu})+\boldsymbol{C}$$

注：若 $\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$ 在 $\boldsymbol{D}$ 上 $\boldsymbol{C}$-类凸，则 $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{D},\boldsymbol{\mu})+\boldsymbol{C}$ 为凸集，其中 $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{D},\boldsymbol{\mu})=\bigcup\limits_{\boldsymbol{y}\in\boldsymbol{D}}\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu})$.

2　(PSVEP)近似解映射 Lipschitz 连续性

本部分讨论(PSVEP)的近似解映射的 Lipschitz连续性，首先建立以下引理.

引理 1　设 $(\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)\in\boldsymbol{\Lambda}\times\boldsymbol{M}$ 给定，$\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)\subset\boldsymbol{\Lambda}\times\boldsymbol{M}$ 为 $(\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)$ 的邻域，假设对任何的 $\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$，有 $\boldsymbol{F}(\cdot,\boldsymbol{y},\boldsymbol{\mu})$ 在 $\boldsymbol{U}(\boldsymbol{\lambda}_0)$ 上为 $\boldsymbol{C}$-凹($\boldsymbol{C}$-凸)，则对任何的 $\boldsymbol{g}\in\boldsymbol{B}_e^*$，有 $\varphi_{\boldsymbol{g}}(\cdot,\boldsymbol{y},\boldsymbol{\mu})=\inf\limits_{w\in\boldsymbol{F}(\cdot,\boldsymbol{y},\boldsymbol{\mu})}\boldsymbol{g}(\boldsymbol{w})$ 在 $\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$ 上为 R_+-凹(R_+-凸).

证明　任取 $\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$ 及 $t\in[0,1]$，$\boldsymbol{w}\in\boldsymbol{F}(t\boldsymbol{x}_1+(1-t)\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})$，由 $\boldsymbol{F}(\cdot,\boldsymbol{y},\boldsymbol{\mu})$ 在 $\boldsymbol{U}(\boldsymbol{\lambda}_0)$ 上为 $\boldsymbol{C}$-凹，则存在 $w_1\in\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{y},\boldsymbol{\mu})$，

$w_2\in F(x_2,y,\mu)$及$c\in C$,使

$$w=tw_1+(1-t)w_2+c$$

故

$$g(w)=g(tw_1+(1-t)w_2+c)$$

由$g\in B_e^*\subset C^*$,知$g\in Y^*$,则g为线性泛函,且

$$g(w)=tg(w_1)+(1-t)g(w_2)+g(c)\geqslant tg(w_1)+(1-t)g(w_2)$$

又由w的任意性知

$$\inf_{w\in F(tx_1+(1-t)x_2,y,\mu)} g(w)\geqslant tg(w_1)+(1-t)g(w_2)\geqslant t(\inf_{w_1\in F(x_1,y,\mu)} g(w_1))+(1-t)(\inf_{w_2\in F(x_2,y,\mu)} g(w_2))$$

因此,

$$\varphi_g(tx_1+(1-t)x_2,y,\mu)\geqslant t\varphi_g(x_1,y,\mu)+(1-t)\varphi_g(x_2,y,\mu)$$

从而$\varphi_g(\cdot,y,\mu)=\inf_{w\in F(\cdot,y,\mu)} g(w)$在$K(U(\lambda_0))$上为$R_+$-凹.

同理可证:$\varphi_g(\cdot,y,\mu)=\inf_{w\in F(\cdot,y,\mu)} g(w)$在$K(U(\lambda_0))$上为$R_+$-凸.

引理2 设$(\lambda_0,\mu_0)\in\Lambda\times M$给定,$l_1,l_2>0$,$U(\lambda_0)\times V(\mu_0)\subset\Lambda\times M$为$(\lambda_0,\mu_0)$的邻域,

(1)若对每个$x,y\in K(U(\lambda_0))$,$F(x,y,\cdot)$在$\mu_0\in M$处关于$e\in\operatorname{int}(C)$为$l_1$-Lipschitz一致连续,则对任何$g\in B_e^*$,有$\varphi_g(x,y,\cdot)=\inf_{w\in F(x,y,\cdot)} g(w)$在$\mu_0\in M$处为$l_1$-Lipschitz一致连续.

(2)若对每个$x\in K(U(\lambda_0))$及$\mu\in V(\mu_0)$,$F(x,\cdot,\mu)$在$K(U(\lambda_0))$上关于$e\in\operatorname{int}(C)$为$l_2$-Lipschitz一致连续,则$\varphi_g(x,\cdot,\mu)=\inf_{w\in F(x,\cdot,\mu)} g(w)$在$K(U(\lambda_0))$上为$l_2$-Lipschitz一致连续.

证明 (1)由假设存在μ_0的邻域$V(\mu_0)$,使得对任何$\mu_1,\mu_2\in V(\mu_0)$及$x,y\in K(U(\lambda_0))$,有

$$F(x,y,\mu_1)\subset F(x,y,\mu_2)+l_1\|\mu_1-\mu_2\|B_0$$

故对任何$w_1\in F(x,y,\mu_1)$,存在$w_2\in F(x,y,\mu_2)$,使得

$$w_1\in w_2+l_1\|\mu_1-\mu_2\|B_0$$

据g的线性性知

$$|g(w_1)-g(w_2)|\leqslant l_1\|\mu_1-\mu_2\|\sup\{g(e):e\in B_0\}\leqslant l_1\|\mu_1-\mu_2\|$$

据μ_1、μ_2的对称性知,

$$|\varphi_g(x,y,\mu_1)-\varphi_g(x,y,\mu_2)|=|\inf_{w\in F(x,y,\mu_1)} g(w)-\inf_{w\in F(x,y,\mu_2)} g(w)|\leqslant l_1\|\mu_1-\mu_2\|$$

结论得证.

(2)类似于(1),可知$\varphi_g(x,\cdot,\mu)=\inf_{w\in F(x,\cdot,\mu)} g(w)$在$K(U(\lambda_0))$上为$l_2$-Lipschitz一致连续.

引理3[18] 对每个$(\varepsilon,\lambda,\mu)\in[\varepsilon_0,+\infty)\times U(\lambda_0)\times V(\mu_0)\subset R_+\times\Lambda\times M$,$x\in K(U(\lambda_0))$,$F(x,\cdot,\mu)$在$K(U(\lambda_0))$上为$C$-类凸,即$F(x,K(U(\lambda_0)),\mu)+C$为凸集,则

$$S_w(\varepsilon,\lambda,\mu)=\bigcup_{f\in C^*\backslash 0} S_f(\varepsilon,\lambda,\mu)=\bigcup_{f\in B_e^*} S_f(\varepsilon,\lambda,\mu)$$

假设$(\varepsilon_0,\lambda_0,\mu_0)\in R_+\times\Lambda\times M$给定,对每个$f\in B_e^*$,存在$(\varepsilon_0,\lambda_0,\mu_0)$的邻域$[\varepsilon_0,+\infty)\times U(\lambda_0)\times V(\mu_0)\subset R_+\times\Lambda\times M$,满足

(A1)$K(\cdot)$在$U(\lambda_0)$上为l_1-Lipschitz连续,即对任何的$\lambda_1,\lambda_2\in U(\lambda_0)$,有

$$K(\lambda_1)\subset K(\lambda_2)+l_1\|\lambda_1-\lambda_2\|B_0$$

(A2)对每个$y\in K(U(\lambda_0))$及$\mu\in V(\mu_0)$,$F(\cdot,y,\mu)$在$K(U(\lambda_0))$上为C-凹;

(A3)对每个$x\in K(U(\lambda_0))$及$\mu\in V(\mu_0)$,$F(x,\cdot,\mu)$在$K(U(\lambda_0))$上关于$e\in\operatorname{int}(C)$为$l_2$-Lipschitz一致连续;

(A4)对每个$x,y\in K(U(\lambda_0))$,$F(x,y,\cdot)$在$\mu_0\in M$处关于$e\in\operatorname{int}(C)$为$l_3$-Lipschitz一致连续;

(A5)对任何$(\varepsilon,\lambda,\mu)\in[\varepsilon_0,+\infty)\times U(\lambda_0)\times V(\mu_0)$,$S_w(\varepsilon,\lambda,\mu)\neq\varnothing$;

(A6)对每个$x\in K(U(\lambda_0))$及$\mu\in V(\mu_0)$,$F(x,\cdot,\mu)$在$K(U(\lambda_0))$上为C-类凸.

设$\rho_0:=\operatorname{diam}(K(\lambda_0))+2l_1(\sup_{\lambda\in U(\lambda_0)}\|\lambda-\lambda_0\|)$.

定理1 假设(A1)与(A2)成立,对任何$\varepsilon_1,\varepsilon_2\in[\varepsilon_0,+\infty)$,$(\lambda,\mu)\in U(\lambda_0)\times V(\mu_0)$,有

$$H(S_f(\varepsilon_1,\lambda,\mu),S_f(\varepsilon_2,\lambda,\mu))\leqslant L_1|\varepsilon_1-\varepsilon_2|$$

其中$L_1=l_0=\rho_0/\varepsilon_0>0$.

证明 设$0<\varepsilon_1<\varepsilon_2$,满足$\varepsilon_2>\varepsilon_0$及任何的$(\lambda,\mu)\in U(\lambda_0)\times V(\mu_0)$,有

$$H(S_f(\varepsilon_1,\lambda,\mu),S_f(\varepsilon_2,\lambda,\mu))\leqslant L_1|\varepsilon_1-\varepsilon_2| \tag{1}$$

其中$L_1=l_0=\rho_0/\varepsilon_0$.

据$S_f(\cdot,\cdot,\cdot)$的定义知

$$S_f(\varepsilon_1,\lambda,\mu)\subset S_f(\varepsilon_2,\lambda,\mu),\quad\forall(\lambda,\mu)\in U(\lambda_0)\times V(\mu_0)$$

则

$$H^*(\boldsymbol{S}_f(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))=0 \tag{2}$$

对每个 $\boldsymbol{x}_2\in\boldsymbol{S}_f(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu})$，$\boldsymbol{x}_0\in\boldsymbol{S}_f(0,\boldsymbol{\lambda},\boldsymbol{\mu})$ 及 $\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$，有

$$\varphi_f(\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})+\varepsilon_2\geqslant 0,\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\geqslant 0$$

故对 $\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$，有

$$\frac{\varepsilon_1}{\varepsilon_2}\varphi_f(\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})+\varepsilon_1+\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2}\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\geqslant 0$$

据(A2)及引理1知

$$\varphi\left(\frac{\varepsilon_1}{\varepsilon_2}\boldsymbol{x}_2+\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2}\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}\right)+\varepsilon_1\geqslant 0$$

故

$$\boldsymbol{x}_1:=\frac{\varepsilon_1}{\varepsilon_2}\boldsymbol{x}_2+\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2}\boldsymbol{x}_0\in\boldsymbol{S}_f(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu})$$

则

$$\|\boldsymbol{x}_2-\boldsymbol{x}_1\|=\frac{|\varepsilon_2-\varepsilon_1|}{\varepsilon_2}\|\boldsymbol{x}_2-\boldsymbol{x}_0\|$$

又 $\boldsymbol{K}(\cdot)$ 在 $\boldsymbol{U}(\boldsymbol{\lambda}_0)$ 上为 l_1-Lipschitz 连续，即 $\boldsymbol{K}(\boldsymbol{\lambda})\subset\boldsymbol{K}(\boldsymbol{\lambda}_0)+l_1\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_0\|\boldsymbol{B}_0$，故 $\mathrm{diam}(\boldsymbol{K}(\boldsymbol{\lambda}))\leqslant\rho_0$，则 $\|\boldsymbol{x}_2-\boldsymbol{x}_1\|\leqslant\frac{\rho_0}{\varepsilon_2}|\varepsilon_2-\varepsilon_1|$，因此

$$H^*(\boldsymbol{S}_f(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant\frac{\rho_0}{\varepsilon_2}|\varepsilon_1-\varepsilon_2| \tag{3}$$

据式(2)、(3)，式(1)成立. 由 $\varepsilon_2\in[\varepsilon_0,+\infty)$，对于式(1)有

$$H(\boldsymbol{S}_f(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant \frac{\rho_0}{\varepsilon_2}|\varepsilon_1-\varepsilon_2|\leqslant\frac{\rho_0}{\varepsilon_0}|\varepsilon_1-\varepsilon_2|=L_1|\varepsilon_1-\varepsilon_2|$$

其中 $L_1=\rho_0/\varepsilon_0=l_0$.

定理2　假设(A1)～(A3)成立，对任意的 $\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$ 及 $(\varepsilon,\boldsymbol{\mu})\in[\varepsilon_0,+\infty)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

其中 $L_2=l_0l_1l_2$.

证明　对任何的 $\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2\in\boldsymbol{U}(\boldsymbol{\lambda}_0)(\boldsymbol{\lambda}_1\neq\boldsymbol{\lambda}_2)$ 及 $(\varepsilon,\boldsymbol{\mu})\in[\varepsilon_0,+\infty)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

其中 $L_2>0$ 且 $L_2=l_0l_1l_2$.

情形1　若 $\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\leqslant\varepsilon/l_1l_2$，设 $a=l_1l_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$，则 $0<a\leqslant\varepsilon$，可断言

$$\boldsymbol{S}_f(\varepsilon-a,\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu})$$

事实上，对任何的 $\boldsymbol{x}^*\in\boldsymbol{S}(\varepsilon-a,\boldsymbol{\lambda}_1,\boldsymbol{\mu})$ 及 $\boldsymbol{y}_2\in\boldsymbol{K}(\boldsymbol{\lambda}_2)$，存在 $\boldsymbol{y}_1\in\boldsymbol{K}(\boldsymbol{\lambda}_1)$，有

$$\|\boldsymbol{y}_1-\boldsymbol{y}_2\|\leqslant l_1\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

且

$$\varphi_f(\boldsymbol{x}^*,\boldsymbol{y}_2,\boldsymbol{\mu})+\varphi_f(\boldsymbol{x}^*,\boldsymbol{y}_1,\boldsymbol{\mu})-\varphi_f(\boldsymbol{x}^*,\boldsymbol{y}_2,\boldsymbol{\mu})+\varepsilon-a\geqslant 0$$

由 $\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$ 在 $\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$ 上关于 $\boldsymbol{e}\in\mathrm{int}(\boldsymbol{C})$ 为 l_2-Lipschitz 一致连续及引理2的(2)知，对任何 $\boldsymbol{y}_2\in\boldsymbol{K}(\boldsymbol{\lambda}_2)$，有

$$\varphi_f(\boldsymbol{x}^*,\boldsymbol{y}_2,\boldsymbol{\mu})+\varepsilon\geqslant 0$$

即 $\boldsymbol{x}^*\in\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，故

$$\boldsymbol{S}_f(\varepsilon-a,\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}) \tag{4}$$

据式(4)及定理1知

$$\begin{aligned}&H^*(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant\\&H^*(\boldsymbol{S}_f(\varepsilon-a,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant\\&H(\boldsymbol{S}_f(\varepsilon-a,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant\\&\frac{\rho_0l_1l_2}{\varepsilon}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\leqslant\frac{\rho_0l_1l_2}{\varepsilon_0}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|=\\&l_0l_1l_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|=L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\end{aligned}$$

其中 $L_2=l_0l_1l_2$.

类似地，

$$H^*(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}))\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

因此

$$H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

情形2　若 $\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|>\varepsilon/l_1l_2$，设 $b=\varepsilon_0/l_1l_2$，$b_0=\mathrm{diam}(\boldsymbol{U}(\boldsymbol{\lambda}_0))<+\infty$，存在充分大的自然数 n_0，使得 $b_0/n_0\leqslant b$，对区间 $[\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2]$ 进行分割：

$$[\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2]:=\{\boldsymbol{\lambda}\in\boldsymbol{U}(\boldsymbol{\lambda}_0):\boldsymbol{\lambda}=\theta\boldsymbol{\lambda}_1+(1-\theta)\boldsymbol{\lambda}_2,\ \theta\in[0,1]\}$$

使得 $\boldsymbol{t}_i=\frac{i-1}{n_0}\boldsymbol{\lambda}_2+\left(1-\frac{i-1}{n_0}\right)\boldsymbol{\lambda}_1$，$i=1,2,\cdots,n_0+1$，且 $\|\boldsymbol{t}_i-\boldsymbol{t}_{i+1}\|=\frac{\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|}{n_0}$，故

$$\|\boldsymbol{t}_i-\boldsymbol{t}_{i+1}\|=\frac{\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|}{n_0}\leqslant\frac{b_0}{n_0}\leqslant b=\frac{\varepsilon_0}{l_1l_2}\leqslant\frac{\varepsilon}{l_1l_2}$$

运用情形1的结论，有

$$\begin{aligned}&H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant\\&\sum_{i=1}^{n_0}H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{t}_i,\boldsymbol{\mu}),\boldsymbol{S}_f(\varepsilon,\boldsymbol{t}_{i+1},\boldsymbol{\mu}))\leqslant\\&L_2\sum_{i=1}^{n_0}\|\boldsymbol{t}_i-\boldsymbol{t}_{i+1}\|=L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\end{aligned}$$

定理3　假设(A1)、(A2)与(A4)成立，对任意的 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{V}(\boldsymbol{\lambda}_0)$ 及 $(\varepsilon,\boldsymbol{\lambda})\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\mu}_0)$，有

$$H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

其中 $L_3=l_0l_3$.

证明　对任何的 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{V}(\boldsymbol{\lambda}_0)(\boldsymbol{\mu}_1\neq\boldsymbol{\mu}_2)$ 及 $(\varepsilon,\boldsymbol{\lambda})\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\mu}_0)$,有

$$(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

情形 1　若 $\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\leqslant\varepsilon/l_3$,设 $a_0=l_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$,则 $0<a_0\leqslant\varepsilon$,故

$$\boldsymbol{S}_f(\varepsilon-a_0,\boldsymbol{\lambda},\boldsymbol{\mu}_1)\subset\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2)$$

事实上,设 $\boldsymbol{x}_0\in\boldsymbol{S}_f(\varepsilon-a_0,\boldsymbol{\lambda},\boldsymbol{\mu}_1)$,则对任何 $\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$,有

$$\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_2)+\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_1)-\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_2)+\varepsilon-a_0\geqslant 0$$

据(A4)及引理 2 的(1)知,

$$|\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_2)-\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_1)|\leqslant l_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|=a_0$$

则 $\varphi_f(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu}_2)+\varepsilon\geqslant 0,\forall\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$,即 $\boldsymbol{x}_0\in\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2)$.类似定理 1 的过程有

$$\begin{aligned}&H^*(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant\\&H^*(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon-a_0,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant\\&H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon-a_0,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant\\&\frac{\rho_0 l_3}{\varepsilon_0}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|=l_0l_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|=L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

其中 $L_3=l_0l_3$.

情形 2　若 $\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|>\varepsilon/l_3$,设 $b=\varepsilon_0/l_3$,$b_0=\mathrm{diam}(\boldsymbol{U}(\boldsymbol{\lambda}_0))<+\infty$,存在充分大的自然数 n_0,使得 $b_0/n_0\leqslant b$,对区间$[\boldsymbol{\mu}_1,\boldsymbol{\mu}_2]$进行分割:

$$[\boldsymbol{\mu}_1,\boldsymbol{\mu}_2]:=\{\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0):\boldsymbol{\mu}=\theta\boldsymbol{\mu}_1+(1-\theta)\boldsymbol{\mu}_2,\ \theta\in[0,1]\}$$

使得 $\boldsymbol{s}_i=\dfrac{i-1}{n_0}\boldsymbol{\mu}_2+\left(1-\dfrac{i-1}{n_0}\right)\boldsymbol{\mu}_1,i=1,2,\cdots,n_0+1$,且 $\|\boldsymbol{s}_i-\boldsymbol{s}_{i+1}\|=\dfrac{\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|}{n_0}$,故

$$\|\boldsymbol{s}_i-\boldsymbol{s}_{i+1}\|=\frac{\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|}{n_0}\leqslant\frac{b_0}{n_0}\leqslant b=\frac{\varepsilon_0}{l_3}\leqslant\frac{\varepsilon}{l_3}$$

运用情形 1 的结论,有

$$\begin{aligned}&H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant\\&\sum_{i=1}^{n_0}H(\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{s}_i),\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{s}_{i+1}))\leqslant\\&L_3\sum_{i=1}^{n_0}\|\boldsymbol{s}_i-\boldsymbol{s}_{i+1}\|=L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

定理 4　假设(A1)～(A6)成立,对任何$(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\begin{aligned}&H(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant\\&L_1|\varepsilon_1-\varepsilon_2|+L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|+L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

其中 $L_1=l_0,L_2=l_0l_1l_2,L_3=l_0l_3$.

证明　注意到(A5)与(A6),据引理 3 知,$\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\bigcup\limits_{f\in\boldsymbol{B}_e^*}\boldsymbol{S}_f(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$.对任何的 $\varepsilon_1,\varepsilon_2\in[\varepsilon_0,+\infty)$及$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,据定理 1 可知

$$H(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant L_1|\varepsilon_1-\varepsilon_2|$$

事实上,任取$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及任何 $\varepsilon_1,\varepsilon_2\in[\varepsilon_0,+\infty)$,只须证对任何的 $\boldsymbol{x}_1\in\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu})=\bigcup\limits_{f\in\boldsymbol{B}_e^*}\boldsymbol{S}_f(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu})$,则存在 $\boldsymbol{f}_0\in\boldsymbol{B}_e^*$,使得

$$\boldsymbol{x}_1:=\boldsymbol{x}(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{S}_{f_0}(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu})$$

类似于定理 1 的过程知,存在 $\boldsymbol{x}_2:=\boldsymbol{x}(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{S}_{f_0}(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu})$,使得

$$d(\boldsymbol{x}(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{x}(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant L_1|\varepsilon_1-\varepsilon_2|$$

故

$$H^*(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant L_1|\varepsilon_1-\varepsilon_2|$$

类似地,有

$$H^*(\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant L_1|\varepsilon_1-\varepsilon_2|$$

从而

$$H(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda},\boldsymbol{\mu}),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda},\boldsymbol{\mu}))\leqslant L_1|\varepsilon_1-\varepsilon_2|\tag{5}$$

类似地,据定理 2 及定理 3 可知,对任何的 $\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$及$(\varepsilon,\boldsymbol{\mu})\in[\varepsilon_0,+\infty)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$H(\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

其中 $L_2=l_0l_1l_2$.

对任何的 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{V}(\boldsymbol{\mu}_0)$及$(\varepsilon,\boldsymbol{\lambda})\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\mu}_0)$,有

$$H(\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

其中 $L_3=l_0l_3$.

据定理 1～3 知,对任何$(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\begin{aligned}&H(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant\\&H(\boldsymbol{S}_w(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1))+\\&H(\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_1))+\\&H(\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_1),\boldsymbol{S}_w(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant\\&L_1|\varepsilon_1-\varepsilon_2|+L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|+L_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

其中 $L_1=l_0,L_2=l_0l_1l_2,L_3=l_0l_3,l_0=\rho_0/\varepsilon_0$.

3　向量优化问题

设 $\boldsymbol{h}:\boldsymbol{D}\times\boldsymbol{M}\subset\boldsymbol{X}\times\boldsymbol{Z}\to\boldsymbol{Y},\boldsymbol{K}:\boldsymbol{\Lambda}\subset\boldsymbol{W}\to 2^{\boldsymbol{D}}\backslash\{\varnothing\}$,对每个$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{\Lambda}\times\boldsymbol{M}$,考虑含参集值向量优化问

题(PVOP)：找 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{\lambda})$，使得

$$\boldsymbol{h}(\boldsymbol{y},\boldsymbol{\mu})-\boldsymbol{h}(\boldsymbol{x}_0,\boldsymbol{\mu})\notin-\boldsymbol{\Omega},\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})$$

其中 $\boldsymbol{\Omega}\cup\boldsymbol{0}$ 为 $\boldsymbol{Y}$ 中的凸锥.

对每个$(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\in R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$，记(PVOP)的所有近似弱有效解的全体为 $\boldsymbol{W}_v(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$，即

$$\boldsymbol{W}_v(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{\lambda}):\boldsymbol{h}(\boldsymbol{y},\boldsymbol{\mu})-\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu})+\varepsilon\cdot\boldsymbol{e}\notin-\mathrm{int}(\boldsymbol{C}),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})\}$$

对 $\boldsymbol{g}\in\boldsymbol{B}_e^*$ 及 $(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\in R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$，记(PVOP)的所有 $\boldsymbol{g}$-近似解的全体为 $\boldsymbol{W}_g(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$，即

$$\boldsymbol{W}_g(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{\lambda}):\boldsymbol{g}(\boldsymbol{h}(\boldsymbol{y},\boldsymbol{\mu}))+\varepsilon\geqslant\boldsymbol{g}(\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu})),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{\lambda})\}$$

定义 4　设 $\boldsymbol{D}\subset\boldsymbol{X}$ 为非空凸子集，对每个$\boldsymbol{\mu}\in\boldsymbol{M}$，称 $\boldsymbol{h}(\cdot,\boldsymbol{\mu})$在 $\boldsymbol{D}$ 上为 $\boldsymbol{C}$-凸(或 $\boldsymbol{C}$-凹)当且仅当对任何的 $\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{D}$ 及 $t\in[0,1]$，有

$$t\boldsymbol{h}(\boldsymbol{x}_1,\boldsymbol{\mu})+(1-t)\boldsymbol{h}(\boldsymbol{x}_2,\boldsymbol{\mu})-\boldsymbol{h}(t\boldsymbol{x}_1+(1-t)\boldsymbol{x}_2,\boldsymbol{\mu})\in\boldsymbol{C}$$

(或 $t\boldsymbol{h}(\boldsymbol{x}_1,\boldsymbol{\mu})+(1-t)\boldsymbol{h}(\boldsymbol{x}_2,\boldsymbol{\mu})-\boldsymbol{h}(t\boldsymbol{x}_1+(1-t)\boldsymbol{x}_2,\boldsymbol{\mu})\in-\boldsymbol{C}$)

定义 5　设 $L>0$，$\boldsymbol{h}:\boldsymbol{D}\times\boldsymbol{M}\subset\boldsymbol{X}\times\boldsymbol{Z}\to\boldsymbol{Y}$，称 $\boldsymbol{h}$ 在 $\boldsymbol{\mu}_0\in\boldsymbol{M}$ 处关于 $\boldsymbol{D}$ 为 L-Lipschitz 一致连续，当且仅当存在 $\boldsymbol{\mu}_0$ 的凸邻域 $\boldsymbol{V}(\boldsymbol{\mu}_0)\subset\boldsymbol{M}$，使对任何 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{V}(\boldsymbol{\mu}_0)$及 $\boldsymbol{x}\in\boldsymbol{D}$，有

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu}_1)\subset\boldsymbol{h}(\boldsymbol{x},\boldsymbol{\mu}_2)+L\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\boldsymbol{B}_0$$

据引理 3 易知，

引理 4　对每个 $\boldsymbol{\mu}\in\boldsymbol{M}$，$\boldsymbol{D}\subset\boldsymbol{X}$ 为非空凸子集，$\boldsymbol{h}(\cdot,\boldsymbol{\mu})$在 $\boldsymbol{D}$ 上为 $\boldsymbol{C}$-凸，则

$$\boldsymbol{W}_v(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\bigcup_{\boldsymbol{g}\in\boldsymbol{C}^*\setminus\boldsymbol{0}}\boldsymbol{W}_g(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})=\bigcup_{\boldsymbol{g}\in\boldsymbol{B}_e^*}\boldsymbol{W}_g(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})$$

假设$(\varepsilon_0,\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)\in R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$ 给定，对每个$\boldsymbol{g}\in\boldsymbol{B}_e^*$，存在$(\varepsilon_0,\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)$的邻域$[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)\subset R_+\times\boldsymbol{\Lambda}\times\boldsymbol{M}$，满足

(B1)$\boldsymbol{K}(\cdot)$在 $\boldsymbol{U}(\boldsymbol{\lambda}_0)$上为 k_1-Lipschitz 连续；

(B2)对每个 $\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{h}(\cdot,\boldsymbol{\mu})$在 $\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$上为 $\boldsymbol{C}$-凹；

(B3)对每个 $\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{h}(\cdot,\boldsymbol{\mu})$在 $\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$上关于 $\boldsymbol{e}\in\mathrm{int}(\boldsymbol{C})$为 k_2-Lipschitz 一致连续；

(B4)对每个 $\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$，$\boldsymbol{h}(\boldsymbol{x},\cdot)$在 $\boldsymbol{M}$ 上关于 $\boldsymbol{e}\in\mathrm{int}(\boldsymbol{C})$为 k_3-Lipschitz 一致连续；

(B5)对任何$(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{W}_v(\varepsilon,\boldsymbol{\lambda},\boldsymbol{\mu})\neq\varnothing$；

(B6)对每个 $\boldsymbol{\mu}\in\boldsymbol{V}(\boldsymbol{\mu}_0)$，$\boldsymbol{h}(\cdot,\boldsymbol{\mu})$在 $\boldsymbol{K}(\boldsymbol{U}(\boldsymbol{\lambda}_0))$上为 $\boldsymbol{C}$-凸的，记 $w_0:=\mathrm{diam}(\boldsymbol{K}(\boldsymbol{\lambda}_0))+2k_1\times(\sup\limits_{\boldsymbol{\lambda}\in\boldsymbol{U}(\boldsymbol{\lambda}_0)}\|\boldsymbol{\lambda}-\boldsymbol{\lambda}_0\|)$.

据引理 4 及定理 4 知，

定理 5　若(B1)～(B6)满足，对任意$(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1)$，$(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in[\varepsilon_0,+\infty)\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$H(\boldsymbol{W}_v(\varepsilon_1,\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{W}_v(\varepsilon_2,\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant K_1|\varepsilon_1-\varepsilon_2|+K_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|+K_3\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

其中 $K_1=k_0,K_2=k_0k_1k_2,K_3=k_0k_3,k_0=w_0/\varepsilon_0$.

4　结　语

本文在文献[1,3,5,16]研究的基础上对含参集值向量均衡问题近似解映射的 Lipschitz 连续性进行了研究.于赋范线性空间中在不具任何单调性的适当条件下，运用标量化技巧，证明了含参集值向量均衡问题近似解映射的 Lipschitz 连续的充分性定理，并将此理论应用于向量优化问题，分析了含参集值向量优化问题 Lipschitz 连续的最优性条件.

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Lipschitz continuity of approximate solution mapping for parametric set-valued vector equilibrium problems

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Abstract: The parametric set-valued vector equilibrium problems are studied in normed linear space. On the basis of introducing the approximate efficient solution of parametric set-valued vector equilibrium problems, the Lipschitz continuity of approximate solution mapping for parametric set-valued vector equilibrium problems is discussed. Sufficient theorem of the Lipschitz continuity of the approximate solution mapping for parametric set-valued vector equilibrium problems is established by using a scalarization method. As applications of these results, the Lipschitz continuity of approximate solution mapping for parametric set-valued vector optimization problem is studied, and sufficient conditions of the Lipschitz continuity of approximate solution mapping for parametric set-valued vector optimization problem are gained.

Key words: Lipschitz continuity; set-valued vector equilibrium problem; approximate efficient solutions; vector optimization problem