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含参集值向量拟均衡问题和对偶问题解 Lipschitz 连续性

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摘要: 在赋范线性空间中研究一类含参集值向量拟均衡问题和对偶问题解的Lipschitz连续性.提出含参集值向量拟均衡问题和对偶问题解的概念,在约束函数具有Lipschitz一致连续性基本假设条件下,运用分析方法建立含参集值向量拟均衡问题和对偶问题解的Lipschitz连续的充分性定理,并给出适当的例子来说明所得结果的有效性.借助理论成果可进一步研究含参集值向量拟均衡问题解的连通性、对偶性及近似计算等.

关键词: 含参集值向量拟均衡问题;解;集值映射;Lipschitz 连续性

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0 引 言

集值向量拟均衡问题是变分不等式、优化问题、交通问题、不动点问题等的统一模型[1-2].集值向量拟均衡问题解的稳定性分析是最优化理论和应用中的一个重要课题.稳定性蕴含了各类连续性,如半连续性[3-4]、连续性[5-6]、Hausdorff 连续性[7-8]等.集值向量拟均衡问题解的稳定性成果不仅可以为集值向量拟均衡问题解的连通性、对偶性、近似计算研究奠定坚实的理论分析依据,而且广泛应用于数理经济、资源配置、交通网络、管理决策及工程设计等众多领域.

(参数)集值向量拟均衡问题解的 Hölder 连续性[9-11]和 Lipschitz 连续性[12-16]是近年来学者们研究的热点课题,对其研究的核心方法主要涉及标量化方法与非线性尺度化方法.借助标量化方法,Peng 等在文献[9]中获得了一类参数集值映射弱广义 Ky Fan 不等式近似解映射的 Hölder 连续的充分性条件;Lam 等在文献[10]中建立了参数向量均衡问题集值近似解映射的 Hölder 连续的充分性定理.运用非线性尺度化方法,Wangkeeree等在文献[11]中得到了具集值映射参数广义向量拟均衡问题解映射的 Hölder 连续的最优性条件.结合标量化技术,Li 等在文献[12]中借助 Hausdorff 度量的概念,建立了有关向量均衡问题近似解映射的 Lipschitz 连续性定理,并将其应用于最优化问题和参数变分不等式;Sadeqi 等在文献[13]中研究了参数集值向量均衡问题近似解映射的 Lipschitz 连续性;Han 在文献[14]中给出了参数广义向量均衡问题强有效近似解映射的 Lipschitz 连续的最优性条件;孟旭东等在文献[15]中分析了含参集值向量均衡问题近似解映射的 Lipschitz 连续性,并应用于含参集值向量优化问题近似解映射的 Lipschitz 连续的充分性条件;在文献[16]中借助向量函数的强凸(凹)性和单调性,应用分析方法建立了参数强向量原始与对偶均衡问题解映射 Lipschitz 连续的充分性定理;在文献[17]中获得了含参向量优化问题的弱解映射、解映射、弱最优值映射及最优值映射的上 Lipschitz 连续和下Lipschitz连续的充分性定理.万德龙等在文献[18]中于解映射不具任何凸性、单调性和单值性的条件下,给出了参数非凸弱广义 Ky Fan 不等式解映射 Lipschitz 连续的充分性条件.本文在赋范线性空间中借助不同于以

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上文献的研究方法研究目标函数和约束函数在参数扰动下两类含参集值向量拟均衡问题和对偶问题解的 Lipschitz 连续性定理.

1 准备知识

设 $\boldsymbol{X}$、$\boldsymbol{Y}$、$\boldsymbol{M}$、$\boldsymbol{N}$ 为赋范线性空间，用 $d(\cdot,\cdot)$ 与 $\|\cdot\|$ 分别表示赋范线性空间中的距离与范数，子集 $\boldsymbol{C}\subset\boldsymbol{Y}$ 且 $\mathrm{int}(\boldsymbol{C})\neq\varnothing$，$\boldsymbol{B}_{\boldsymbol{X}}\subset\boldsymbol{X}$，$\boldsymbol{B}_{\boldsymbol{Y}}\subset\boldsymbol{Y}$ 为闭单位球.

设 $\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}\to 2^{\boldsymbol{Y}}\setminus\{\varnothing\}$，$\boldsymbol{K}:\boldsymbol{X}\times\boldsymbol{M}\to 2^{\boldsymbol{X}}\setminus\{\varnothing\}$ 为集值映射，对每个点 $(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{M}\times\boldsymbol{N}$，研究以下两类含参集值向量拟均衡问题，分别简记为问题(PSVQEP1)与问题(PSVQEP2).

问题(PSVQEP1)：找到点 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\cap(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))\neq\varnothing,\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$$

问题(PSVQEP2)：找到点 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{x}_0,\boldsymbol{y},\boldsymbol{\mu})\subset\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$$

记集合 $\boldsymbol{E}(\boldsymbol{\lambda})=\{\boldsymbol{x}\in\boldsymbol{X}\mid\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$，用 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu})$ 与 $\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu})$ 分别表示问题(PSVQEP1)的解集与问题(PSVQEP2)的解集，即

$$\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})\mid\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu})\cap(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))\neq\varnothing,\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$$

与

$$\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})\mid\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu})\subset\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$$

问题(PSVQEP1)与问题(PSVQEP2)的对偶问题分别记为问题(DPSVQEP1)与问题(DPSVQEP2).

问题(DPSVQEP1)：找到点 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_0,\boldsymbol{\mu})\cap(-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))\neq\varnothing,\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$$

问题(DPSVQEP2)：找到点 $\boldsymbol{x}_0\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$，使得

$$\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_0,\boldsymbol{\mu})\subset-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x}_0,\boldsymbol{\lambda})$$

用 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu})$ 与 $\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu})$ 分别表示问题(DPSVQEP1)的解集与问题(DPSVQEP2)的解集，即

$$\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})\mid\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x},\boldsymbol{\mu})\cap(-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))\neq\varnothing,\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$$

与

$$\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu})=\{\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})\mid\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x},\boldsymbol{\mu})\subset-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})),\ \forall\,\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$$

设 $\boldsymbol{A},\boldsymbol{B}\subset\boldsymbol{X}$ 为非空有界闭子集，记 $\rho(\boldsymbol{A},\boldsymbol{B})=\max\{\sup\limits_{\boldsymbol{a}\in\boldsymbol{A}}d(\boldsymbol{a},\boldsymbol{B}),\sup\limits_{\boldsymbol{b}\in\boldsymbol{B}}d(\boldsymbol{A},\boldsymbol{b})\}$，其中 $d(\boldsymbol{a},\boldsymbol{B})=\inf\limits_{\boldsymbol{b}\in\boldsymbol{B}}\|\boldsymbol{a}-\boldsymbol{b}\|$，$d(\boldsymbol{A},\boldsymbol{b})=\inf\limits_{\boldsymbol{a}\in\boldsymbol{A}}\|\boldsymbol{a}-\boldsymbol{b}\|$.

定义 1[14] 设 $L\geqslant 0$，$\boldsymbol{G}:\boldsymbol{M}\to 2^{\boldsymbol{X}}\setminus\{\varnothing\}$ 为集值映射，称 $\boldsymbol{G}$ 在 $\boldsymbol{M}$ 上为 L-Lipschitz 连续的当且仅当对任意的点 $\boldsymbol{\lambda}_1,\boldsymbol{\lambda}_2\in\boldsymbol{M}$，有

$$\boldsymbol{G}(\boldsymbol{\lambda}_1)\subset\boldsymbol{G}(\boldsymbol{\lambda}_2)+L\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\boldsymbol{B}_{\boldsymbol{X}}$$

定义 2 设 $L\geqslant 0$，$\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}\to 2^{\boldsymbol{Y}}\setminus\{\varnothing\}$ 为集值映射，

(1)称 $\boldsymbol{F}$ 在 $\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}$ 上关于第 1 个变量为 L-Lipschitz 连续的当且仅当对任意的点 $(\boldsymbol{y},\boldsymbol{\mu})\in\boldsymbol{X}\times\boldsymbol{N}$ 及点 $\boldsymbol{x}_1,\boldsymbol{x}_2\in\boldsymbol{X}$，有

$$\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{y},\boldsymbol{\mu})\subset\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})+L\|\boldsymbol{x}_1-\boldsymbol{x}_2\|\boldsymbol{B}_{\boldsymbol{Y}}$$

(2)称 $\boldsymbol{F}$ 在 $\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}$ 上关于第 2 个变量为 L-Lipschitz 连续的当且仅当对任意的点 $(\boldsymbol{x},\boldsymbol{\mu})\in\boldsymbol{X}\times\boldsymbol{N}$ 及点 $\boldsymbol{y}_1,\boldsymbol{y}_2\in\boldsymbol{X}$，有

$$\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y}_1,\boldsymbol{\mu})\subset\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y}_2,\boldsymbol{\mu})+L\|\boldsymbol{y}_1-\boldsymbol{y}_2\|\boldsymbol{B}_{\boldsymbol{Y}}$$

(3)称 $\boldsymbol{F}$ 在 $\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}$ 上关于第 3 个变量为 L-Lipschitz 连续的当且仅当对任意的点 $(\boldsymbol{x},\boldsymbol{y})\in\boldsymbol{X}\times\boldsymbol{X}$ 及点 $\boldsymbol{\mu}_1,\boldsymbol{\mu}_2\in\boldsymbol{N}$，有

$$\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu}_1)\subset\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\boldsymbol{\mu}_2)+L\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\boldsymbol{B}_{\boldsymbol{Y}}$$

定义 3 设 $\alpha\geqslant 0$，$\beta\geqslant 0$，$\boldsymbol{K}:\boldsymbol{X}\times\boldsymbol{M}\to 2^{\boldsymbol{X}}\setminus\{\varnothing\}$ 为集值映射，称 $\boldsymbol{K}$ 在 $\boldsymbol{X}\times\boldsymbol{M}$ 上为 α-β-Lipschitz 一致连续的当且仅当对任意的点 $(\boldsymbol{x}_1,\boldsymbol{\lambda}_1)$，$(\boldsymbol{x}_2,\boldsymbol{\lambda}_2)\in\boldsymbol{X}\times\boldsymbol{M}$，有

$$\boldsymbol{K}(\boldsymbol{x}_1,\boldsymbol{\lambda}_1)\subset\{\boldsymbol{x}\in\boldsymbol{X}\mid\text{存在点 }\boldsymbol{y}\in\boldsymbol{K}(\boldsymbol{x}_2,\boldsymbol{\lambda}_2)\text{，有 }\|\boldsymbol{x}-\boldsymbol{y}\|\leqslant\alpha\|\boldsymbol{x}_1-\boldsymbol{x}_2\|+\beta\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|\}$$

2 问题(PSVQEP1)与问题(PSVQEP2)解的 Lipschitz 连续性

本章研究问题(PSVQEP1)与问题(PSVQEP2)解的 Lipschitz 连续性，为叙述的简洁性，特给出以下基本假设.

设 $\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}\to 2^{\boldsymbol{Y}}\setminus\{\varnothing\}$，$\boldsymbol{K}:\boldsymbol{X}\times\boldsymbol{M}\to$

$2^{\boldsymbol{X}}\backslash\{\varnothing\}$ 为集值映射，$\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)\subset\boldsymbol{M}\times\boldsymbol{N}$ 为给定点$(\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)\in\boldsymbol{M}\times\boldsymbol{N}$ 的邻域，对任意的点 $\boldsymbol{\lambda}\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$，记集合 $\boldsymbol{E}(\boldsymbol{\lambda})=\{\boldsymbol{x}\in\boldsymbol{X}\mid\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$.

(H1)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu})\subset\boldsymbol{E}(\boldsymbol{\lambda})$ 为非空紧子集；

(H2)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有 $\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu})\subset\boldsymbol{E}(\boldsymbol{\lambda})$ 为非空紧子集；

(H3)集值映射 $\boldsymbol{K}(\cdot,\cdot)$ 在 $\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0))\times\boldsymbol{U}(\boldsymbol{\lambda}_0)\subset\boldsymbol{X}\times\boldsymbol{M}$ 上为 L_1-L_2-Lipschitz 一致连续的；

(H4)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{y}\in\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0))\backslash\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu})$，存在点 $\boldsymbol{x}_1\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu})$，有

$$\|\boldsymbol{x}_1-\boldsymbol{y}\|\leqslant\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{y},\boldsymbol{\mu})}d(\boldsymbol{f},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))+\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_1,\boldsymbol{\mu})}d(\boldsymbol{g},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))$$

(H5)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{y}\in\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0))\backslash\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu})$，存在点 $\boldsymbol{x}_2\in\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu})$，有

$$\|\boldsymbol{x}_2-\boldsymbol{y}\|\leqslant\sup_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})}d(\boldsymbol{f},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))+\sup_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_2,\boldsymbol{\mu})}d(\boldsymbol{g},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))$$

(H6)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})$，集值映射 $\boldsymbol{F}(\boldsymbol{x},\cdot,\boldsymbol{\mu})$关于第 2 个变量在 $\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0)),\boldsymbol{U}(\boldsymbol{\lambda}_0))$上为 L_3-Lipschitz 连续的；

(H7)对任意的点 $\boldsymbol{\lambda}\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$及点 $\boldsymbol{x},\boldsymbol{y}\in\boldsymbol{E}(\boldsymbol{\lambda})$，集值映射 $\boldsymbol{F}(\boldsymbol{x},\boldsymbol{y},\cdot)$关于第 3 个变量在 $\boldsymbol{V}(\boldsymbol{\mu}_0)$上为 L_4-Lipschitz 连续的；

(H8)记 $L_{\boldsymbol{\mu}}=\dfrac{L_4}{1-2L_1L_3}$，其中 $0\leqslant L_1L_3<1/2$；

(H9)记 $L_{\boldsymbol{\lambda}}=\dfrac{2L_2L_3}{1-2L_1L_3}$，其中 $0\leqslant L_1L_3<1/2$.

首先讨论问题(PSVQEP1)解的 Lipschitz 连续性.

定理 1　假若条件(H1)、(H3)、(H4)、(H6)、(H7)、(H8)成立，则对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$，$(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_\mu\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\| \tag{1}$$

证明　显然地，若 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)=\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，则式(1)成立.

假设 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\neq\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，分以下两种情形讨论：

情形 1　若 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\not\subset\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$且 $\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\not\subset\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$，对任意的点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\backslash\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，据条件(H4)知，存在点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，有

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2)}d(\boldsymbol{f},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))+\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2)}d(\boldsymbol{g},\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C}))) \tag{2}$$

据点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\lambda})$，$\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\lambda})$，结合条件(H3)知，存在点 $\boldsymbol{x}_1\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\lambda})$，$\boldsymbol{x}_2\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\lambda})$，有

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}_1\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\| \tag{3}$$

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)-\boldsymbol{x}_2\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\| \tag{4}$$

由 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$，$\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，知存在点 $\boldsymbol{y}_1\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}_2,\boldsymbol{\mu}_1)\cap(\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))$，$\boldsymbol{y}_2\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}_1,\boldsymbol{\mu}_2)\cap(\boldsymbol{Y}\backslash(-\operatorname{int}(\boldsymbol{C})))$，结合式(2)，有

$$\begin{aligned}&\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\\&\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2)}d(\boldsymbol{f},\boldsymbol{y}_2)+\\&\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2)}d(\boldsymbol{g},\boldsymbol{y}_1)\leqslant\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}_1,\boldsymbol{\mu}_2))+\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}_2,\boldsymbol{\mu}_1),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2))\leqslant\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}_1,\boldsymbol{\mu}_2))+\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}_2,\boldsymbol{\mu}_1),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_1))+\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_1),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2))\end{aligned}$$

再注意到条件(H6)、(H7)，并结合式(3)与式(4)，得

$$\begin{aligned}&\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant L_3\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}_1\|+\\&L_3\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)-\boldsymbol{x}_2\|+L_4\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\leqslant\\&2L_3L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|+L_4\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

据条件(H8)，知

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\frac{L_4}{1-2L_1L_3}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|=L_\mu\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

又注意到点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\backslash\boldsymbol{S}_1(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$

任意性，有

$$\sup_{x(\lambda,\mu_1)\in S_1(\lambda,\mu_1)\setminus S_1(\lambda,\mu_2)}\inf_{x(\lambda,\mu_2)\in S_1(\lambda,\mu_2)}\|x(\lambda,\mu_1)-x(\lambda,\mu_2)\|\leqslant L_\mu\|\mu_1-\mu_2\| \tag{5}$$

再结合 $d(\cdot,\cdot)$ 的定义，知

$$\sup_{x(\lambda,\mu_1)\in S_1(\lambda,\mu_1)} d(x(\lambda,\mu_1),S_1(\lambda,\mu_2))= \sup_{x(\lambda,\mu_1)\in S_1(\lambda,\mu_1)\setminus S_1(\lambda,\mu_2)} d(x(\lambda,\mu_1),S(\lambda,\mu_2))\leqslant L_\mu\|\mu_1-\mu_2\| \tag{6}$$

类似可得

$$\sup_{x(\lambda,\mu_2)\in S_1(\lambda,\mu_2)} d(S_1(\lambda,\mu_1),x(\lambda,\mu_2))\leqslant L_\mu\|\mu_1-\mu_2\| \tag{7}$$

据式(6)与式(7)知，式(1)成立.

情形 2 若 $S_1(\lambda,\mu_1)\subset S_1(\lambda,\mu_2)$ 或 $S_1(\lambda,\mu_2)\subset S_1(\lambda,\mu_1)$，不失一般性，不妨假设 $S_1(\lambda,\mu_1)\subset S_1(\lambda,\mu_2)$，据 $d(\cdot,\cdot)$ 的定义，得

$$\sup_{x(\lambda,\mu_1)\in S_1(\lambda,\mu_1)} d(x(\lambda,\mu_1),S_1(\lambda,\mu_2))=0 \tag{8}$$

类似情形 1 同样的论证过程知式(7)成立. 结合式(7)与式(8)知，式(1)成立.

定理 2 假若条件(H1)、(H3)、(H4)、(H6)、(H9)成立，则对任意的点 (λ_1,μ)，$(\lambda_2,\mu)\in U(\lambda_0)\times V(\mu_0)$，有

$$\rho(S_1(\lambda_1,\mu),S_1(\lambda_2,\mu))\leqslant L_\lambda\|\lambda_1-\lambda_2\| \tag{9}$$

证明 显然地，若 $S_1(\lambda_1,\mu)=S_1(\lambda_2,\mu)$，则式(9)成立.

假设 $S_1(\lambda_1,\mu)\neq S_1(\lambda_2,\mu)$，分以下两种情形讨论：

情形 1 若 $S_1(\lambda_1,\mu)\not\subset S_1(\lambda_2,\mu)$ 且 $S_1(\lambda_2,\mu)\not\subset S_1(\lambda_1,\mu)$，对任意的点 $x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)\setminus S_1(\lambda_1,\mu)$，据条件(H4)知，存在点 $x(\lambda_1,\mu)\in S_1(\lambda_1,\mu)$，有

$$\|x(\lambda_2,\mu)-x(\lambda_1,\mu)\|\leqslant \inf_{f\in F(x(\lambda_1,\mu),x(\lambda_2,\mu),\mu)} d(f,Y\setminus(-\mathrm{int}(C)))+ \inf_{g\in F(x(\lambda_2,\mu),x(\lambda_1,\mu),\mu)} d(g,Y\setminus(-\mathrm{int}(C))) \tag{10}$$

据点 $x(\lambda_2,\mu)\in K(x(\lambda_2,\mu),\lambda_2)$，$x(\lambda_1,\mu)\in K(x(\lambda_1,\mu),\lambda_1)$，结合条件(H3)知，存在点 $x_1\in K(x(\lambda_2,\mu),\lambda_1)$，$x_2\in K(x(\lambda_1,\mu),\lambda_2)$，有

$$\|x(\lambda_2,\mu)-x_1\|\leqslant L_2\|\lambda_1-\lambda_2\| \tag{11}$$

$$\|x(\lambda_1,\mu)-x_2\|\leqslant L_2\|\lambda_1-\lambda_2\| \tag{12}$$

再由条件(H3)知，存在点 $\bar{x}_1\in K(x(\lambda_1,\mu),\lambda_1)$，$\bar{x}_2\in K(x(\lambda_2,\mu),\lambda_2)$，有

$$\|\bar{x}_1-x_1\|\leqslant L_1\|x(\lambda_1,\mu)-x(\lambda_2,\mu)\| \tag{13}$$

$$\|\bar{x}_2-x_2\|\leqslant L_1\|x(\lambda_1,\mu)-x(\lambda_2,\mu)\| \tag{14}$$

据 $x(\lambda_1,\mu)\in S_1(\lambda_1,\mu)$，$x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)$，知存在点 $y_1\in F(x(\lambda_2,\mu),\bar{x}_2,\mu)\cap(Y\setminus(-\mathrm{int}(C)))$，$y_2\in F(x(\lambda_1,\mu),\bar{x}_1,\mu)\cap(Y\setminus(-\mathrm{int}(C)))$，结合式(10)，有

$$\begin{aligned}
&\|x(\lambda_2,\mu)-x(\lambda_1,\mu)\|\leqslant\\
&\inf_{f\in F(x(\lambda_1,\mu),x(\lambda_2,\mu),\mu)} d(f,y_2)+\\
&\inf_{g\in F(x(\lambda_2,\mu),x(\lambda_1,\mu),\mu)} d(g,y_1)\leqslant\\
&\rho(F(x(\lambda_1,\mu),x(\lambda_2,\mu),\mu),F(x(\lambda_1,\mu),\bar{x}_1,\mu))+\\
&\rho(F(x(\lambda_2,\mu),x(\lambda_1,\mu),\mu),F(x(\lambda_2,\mu),\bar{x}_2,\mu))\leqslant\\
&\rho(F(x(\lambda_1,\mu),x(\lambda_2,\mu),\mu),F(x(\lambda_1,\mu),x_1,\mu))+\\
&\rho(F(x(\lambda_1,\mu),x_1,\mu),F(x(\lambda_1,\mu),\bar{x}_1,\mu))+\\
&\rho(F(x(\lambda_2,\mu),x(\lambda_1,\mu),\mu),F(x(\lambda_2,\mu),x_2,\mu))+\\
&\rho(F(x(\lambda_2,\mu),x_2,\mu),F(x(\lambda_2,\mu),\bar{x}_2,\mu))
\end{aligned}$$

再注意到条件(H6)，并结合式(11)～(14)，得

$$\begin{aligned}
&\|x(\lambda_2,\mu)-x(\lambda_1,\mu)\|\leqslant\\
&L_3\|x(\lambda_2,\mu)-x_1\|+L_3\|x_1-\bar{x}_1\|+\\
&L_3\|x(\lambda_1,\mu)-x_2\|+L_3\|x_2-\bar{x}_2\|\leqslant\\
&2L_3L_1\|x(\lambda_1,\mu)-x(\lambda_2,\mu)\|+\\
&2L_3L_2\|\lambda_1-\lambda_2\|
\end{aligned}$$

据条件(H9)，知

$$\|x(\lambda_2,\mu)-x(\lambda_1,\mu)\|\leqslant \frac{2L_2L_3}{1-2L_1L_3}\|\lambda_1-\lambda_2\|=L_\lambda\|\lambda_1-\lambda_2\|$$

又注意到点 $x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)\setminus S_1(\lambda_1,\mu)$ 任意性，有

$$\sup_{x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)\setminus S_1(\lambda_1,\mu)}\inf_{x(\lambda_1,\mu)\in S_1(\lambda_1,\mu)}\|x(\lambda_2,\mu)-x(\lambda_1,\mu)\|\leqslant L_\lambda\|\lambda_1-\lambda_2\|$$

再结合 $d(\cdot,\cdot)$ 的定义，知

$$\sup_{x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)} d(S_1(\lambda_1,\mu),x(\lambda_2,\mu))= \sup_{x(\lambda_2,\mu)\in S_1(\lambda_2,\mu)\setminus S_1(\lambda_1,\mu)} d(S_1(\lambda_1,\mu),x(\lambda_2,\mu))\leqslant L_\lambda\|\lambda_1-\lambda_2\| \tag{15}$$

类似可证

$$\sup_{x(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} d(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_\lambda\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{16}$$

据式(15)与式(16)知，式(9)成立.

情形 2　若 $\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$ 或 $\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\subset\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，不失一般性，不妨假设 $\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，据 $d(\cdot,\cdot)$ 的定义，得

$$\sup_{x(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} d(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))=0 \tag{17}$$

类似情形 1 的论证过程知式(15)成立. 结合式(15)与式(17)知，式(9)成立.

定理 3　假若条件(H1)、(H3)、(H4)、(H6)、(H7)、(H8)、(H9)成立，则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant L_\mu\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|+L_\lambda\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{18}$$

证明　注意到 $\rho(\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant\rho(\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_2))+\rho(\boldsymbol{S}_1(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_2),\boldsymbol{S}_1(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))$，并结合式(1)与式(9)知，式(18)成立.

现给出例子说明定理 3 中结果的有效性.

例 1　设 $\boldsymbol{X}=\boldsymbol{Y}=\boldsymbol{R}$，$\boldsymbol{M}=\boldsymbol{N}=[0,1]$，$\boldsymbol{C}=\boldsymbol{R}_+=[0,+\infty)$，$\boldsymbol{F}(x,y,\lambda)=[xy-(1+\lambda)y+2,(1+\lambda)x+6]$，$\boldsymbol{K}(x,\lambda)=[((1+\lambda)^2+x)/16,2]$，则 $\boldsymbol{E}(\lambda)=[(1+\lambda)^2/15,2]$. 取 $\lambda_0=\dfrac{1}{2}$，$\boldsymbol{U}(\lambda_0)=\boldsymbol{M}=[0,1]$，对任何的 $\lambda\in\boldsymbol{U}(\lambda_0)$，有 $\boldsymbol{E}(\lambda)=\boldsymbol{E}(\boldsymbol{U}(\lambda_0))=[1/15,2]$，$\boldsymbol{S}_1(\lambda,\lambda)=[1+\lambda,2]$，$\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\backslash\boldsymbol{S}_1(\lambda,\lambda)=[1/15,1+\lambda]$.

易知，对任何的点 $\lambda\in\boldsymbol{U}(\lambda_0)$，有 $\boldsymbol{S}_1(\lambda,\lambda)\subset\boldsymbol{E}(\lambda)$ 为非空紧子集，$\boldsymbol{K}(\cdot,\cdot)$ 在 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\times\boldsymbol{U}(\lambda_0)$ 上为 $\dfrac{1}{16}$-$\dfrac{1}{4}$-Lipschitz 一致连续的，对任意点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x\in\boldsymbol{E}(\lambda)$，$\boldsymbol{F}(x,\cdot,\lambda)$ 在 $\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\lambda_0)),\boldsymbol{U}(\lambda_0))$ 上为 4-Lipschitz 连续的，以及对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x,y\in\boldsymbol{E}(\lambda)$，$\boldsymbol{F}(x,y,\cdot)$ 在 $\boldsymbol{U}(\lambda_0)$ 上为 4-Lipschitz 连续的，且对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $y\in[1/15,1+\lambda]$，取点 $x_1=1+\lambda\in\boldsymbol{S}_1(\lambda,\lambda)$，有

$$\inf_{f\in \boldsymbol{F}(x_1,y,\boldsymbol{\mu})} d(\boldsymbol{f},\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in \boldsymbol{F}(y,x_1,\boldsymbol{\mu})} d(\boldsymbol{g},\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))\geqslant(1+\lambda)\|y-(1+\lambda)\|\geqslant\|x_1-y\|$$

此外，易见 $L_1L_3=\dfrac{1}{16}\cdot 4=\dfrac{1}{4}\in[0,1/2)$.

类似问题(PSVQEP1)解的 Lipschitz 连续性的论证过程，易得问题(PSVQEP2)解的 Lipschitz 连续性定理.

定理 4　假若条件(H2)、(H3)、(H5)、(H6)、(H7)、(H8)成立，则对任意的点 $(\boldsymbol{\lambda},\boldsymbol{\mu}_1),(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_2(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_\mu\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

定理 5　假若条件(H2)、(H3)、(H5)、(H6)、(H9)成立，则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_2(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_2(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_\lambda\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

定理 6　假若条件(H2)、(H3)、(H5)、(H6)、(H7)、(H8)、(H9)成立，则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_2(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_2(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant L_\mu\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|+L_\lambda\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

现举例检验定理 6 结论的有效性.

例 2　设 $\boldsymbol{X}=\boldsymbol{Y}=\boldsymbol{R}$，$\boldsymbol{M}=\boldsymbol{N}=[0,1]$，$\boldsymbol{C}=\boldsymbol{R}_+=[0,+\infty)$，$\boldsymbol{F}(x,y,\lambda)=[xy-(1+\lambda)x-(1+\lambda)y+2,+\infty)$，$\boldsymbol{K}(x,\lambda)=[((1+\lambda)^2+x)/16,2]$，则 $\boldsymbol{E}(\lambda)=[(1+\lambda)^2/15,2]$. 取 $\lambda_0=\dfrac{1}{2}$，$\boldsymbol{U}(\lambda_0)=\boldsymbol{M}=[0,1]$，对任何的 $\lambda\in\boldsymbol{U}(\lambda_0)$，有 $\boldsymbol{E}(\lambda)=\boldsymbol{E}(\boldsymbol{U}(\lambda_0))=[1/15,2]$，$\boldsymbol{S}_2(\lambda,\lambda)=[1+\lambda,2]$，$\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\backslash\boldsymbol{S}_2(\lambda,\lambda)=[1/15,1+\lambda]$.

易知，对任何的 $\lambda\in\boldsymbol{U}(\lambda_0)$，有 $\boldsymbol{S}_2(\lambda,\lambda)\subset\boldsymbol{E}(\lambda)$ 为非空紧子集，$\boldsymbol{K}(\cdot,\cdot)$ 在 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\times\boldsymbol{U}(\lambda_0)$ 上为 $\dfrac{1}{16}$-$\dfrac{1}{4}$-Lipschitz 一致连续的，对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x\in\boldsymbol{E}(\lambda)$，$\boldsymbol{F}(x,\cdot,\lambda)$ 在 $\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\lambda_0)),\boldsymbol{U}(\lambda_0))$ 上为 4-Lipschitz 连续的，以及对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x,y\in\boldsymbol{E}(\lambda)$，$\boldsymbol{F}(x,y,\cdot)$ 在 $\boldsymbol{U}(\lambda_0)$ 上为 4-Lipschitz 连续的，对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$，$y\in[1/15,1+\lambda]$，取点 $x_2=1+\lambda\in\boldsymbol{S}_2(\lambda,\lambda)$，有

$$\inf_{f\in \boldsymbol{F}(x_1,y,\boldsymbol{\mu})} d(\boldsymbol{f},\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in \boldsymbol{F}(y,x_1,\boldsymbol{\mu})} d(\boldsymbol{g},\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))\geqslant(1+\lambda)\|y-(1+\lambda)\|\geqslant\|x_2-y\|$$

此外，易见 $L_1L_3=\dfrac{1}{16}\cdot 4=\dfrac{1}{4}\in[0,1/2)$.

3 问题(DPSVQEP1)与问题(DPSVQEP2)解的 Lipschitz 连续性

本章分析问题(DPSVQEP1)与问题(DPSVQEP2)解的 Lipschitz 连续性,为叙述的方便性,特给出以下基本假设.

设 $\boldsymbol{F}:\boldsymbol{X}\times\boldsymbol{X}\times\boldsymbol{N}\to 2^{Y}\setminus\{\varnothing\}$,$\boldsymbol{K}:\boldsymbol{X}\times\boldsymbol{M}\to 2^{X}\setminus\{\varnothing\}$为集值映射,$\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)\subset\boldsymbol{M}\times\boldsymbol{N}$ 为给定点$(\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)\in\boldsymbol{M}\times\boldsymbol{N}$ 的邻域,对任意的点 $\boldsymbol{\lambda}\in\boldsymbol{U}(\boldsymbol{\lambda}_0)$,记集合 $\boldsymbol{E}(\boldsymbol{\lambda})=\{\boldsymbol{x}\in\boldsymbol{X}\mid\boldsymbol{x}\in\boldsymbol{K}(\boldsymbol{x},\boldsymbol{\lambda})\}$.

(T1)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu})\subset\boldsymbol{E}(\boldsymbol{\lambda})$为非空紧子集;

(T2)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有 $\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu})\subset\boldsymbol{E}(\boldsymbol{\lambda})$为非空紧子集;

(T3)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{y}\in\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0))\setminus\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu})$,存在点 $\boldsymbol{x}_1\in\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu})$,有

$$\|\boldsymbol{x}_1-\boldsymbol{y}\|\leqslant\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{y},\boldsymbol{\mu})}d(\boldsymbol{f},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))+\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_1,\boldsymbol{\mu})}d(\boldsymbol{g},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))$$

(T4)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{y}\in\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0))\setminus\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu})$,存在点 $\boldsymbol{x}_2\in\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu})$,有

$$\|\boldsymbol{x}_2-\boldsymbol{y}\|\leqslant\sup_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{y},\boldsymbol{\mu})}d(\boldsymbol{f},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))+\sup_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{y},\boldsymbol{x}_2,\boldsymbol{\mu})}d(\boldsymbol{g},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))$$

(T5)对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$及点 $\boldsymbol{x}\in\boldsymbol{E}(\boldsymbol{\lambda})$,集值映射 $\boldsymbol{F}(\cdot,\boldsymbol{x},\boldsymbol{\mu})$关于第 1 个变量在$\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\boldsymbol{\lambda}_0)),\boldsymbol{U}(\boldsymbol{\lambda}_0))$上为 L_5-Lipschitz 连续的;

(T6)记 $L_\mu^D=\dfrac{L_4}{1-2L_1L_5}$,其中 $0\leqslant L_1L_5<\dfrac{1}{2}$;

(T7)记 $L_\lambda^D=\dfrac{2L_2L_5}{1-2L_1L_5}$,其中 $0\leqslant L_1L_5<\dfrac{1}{2}$.

首先分析问题(DPSVQEP1)解的 Lipschitz 连续性.

定理 7 假若条件(T1)、(T3)、(H3)、(T5)、(T6)、(H7)成立,则对任意的点$(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$,$(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_\mu^D\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\| \tag{19}$$

证明 显然地,若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)=\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$,则式(19)成立.

假设 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\neq\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$,分以下两种情形讨论:

情形 1 若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\not\subset\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$且 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\not\subset\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$,对任意点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\setminus\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$,据条件(T3)知,存在点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$,有

$$\begin{aligned}&\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\\&\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2)}d(\boldsymbol{f},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))+\\&\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2)}d(\boldsymbol{g},-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))\end{aligned} \tag{20}$$

据点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\lambda})$,$\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\lambda})$,结合条件(H3)知,存在点 $\boldsymbol{x}_1\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\lambda})$,$\boldsymbol{x}_2\in\boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\lambda})$,有

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}_1\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\| \tag{21}$$

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)-\boldsymbol{x}_2\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\| \tag{22}$$

由 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$,$\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$,知存在点 $\boldsymbol{y}_1\in\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_1)\cap(-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))$,$\boldsymbol{y}_2\in\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2)\cap(-(\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C}))))$,结合式(20),有

$$\begin{aligned}&\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\\&\inf_{\boldsymbol{f}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2)}d(\boldsymbol{f},\boldsymbol{y}_1)+\\&\inf_{\boldsymbol{g}\in\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2)}d(\boldsymbol{g},\boldsymbol{y}_2)\leqslant\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_1))+\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2))\leqslant\\&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\\&\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_1))+\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_1),\\&\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{\mu}_1))+\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\\&\boldsymbol{\mu}_2),\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2),\boldsymbol{\mu}_2))\end{aligned}$$

由条件(T5)、(H7),式(21)、式(22),得

$$\begin{aligned}&\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\\&L_4\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|+L_5\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)-\boldsymbol{x}_2\|+\\&L_5\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}_1\|\leqslant\\&2L_5L_1\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|+L_4\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|\end{aligned}$$

并注意到(T6),知

$$\|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant\frac{L_4}{1-2L_1L_5}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|=L_\mu^D\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\| \tag{23}$$

又由点 $\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$ 任意性，有

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)}\ \inf_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)} \|\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)-\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\|\leqslant L_\mu^D\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

再结合 $d(\cdot,\cdot)$ 的定义，知

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)} d(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2))= \sup_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)} d(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_\mu^D\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\| \tag{24}$$

类似可证

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)} d(\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_\mu^D\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\| \tag{25}$$

据式(24)与式(25)知，式(19)成立.

情形 2　若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\subset \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$ 或 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\subset \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$，不失一般性，不妨假设 $\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\subset \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2)$，据 $d(\cdot,\cdot)$ 的定义，得

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1)\in \boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1)} d(\boldsymbol{x}(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2))=0 \tag{26}$$

类似情形 1 的论证过程知式(25)成立. 结合式(25)与式(26)知，式(19)成立.

定理 8　假若条件(T1)、(T3)、(H3)、(T5)、(T7)成立，则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，$(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{U}(\boldsymbol{\lambda}_0)\times \boldsymbol{V}(\boldsymbol{\mu}_0)$，有

$$\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_\lambda^D\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{27}$$

证明　显然地，若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})=\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，则式(27)成立.

假设 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\neq \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，分以下两种情形讨论：

情形 1　若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\not\subset \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，$\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\not\subset \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，对任意的点 $\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，据条件(T3)知，存在点 $\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，有

$$\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\|\leqslant \inf_{f\in \boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu})} d(\boldsymbol{f},-(\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C}))))+ \inf_{g\in \boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu})} d(\boldsymbol{g},-(\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))) \tag{28}$$

再据点 $\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\lambda}_2)$，$\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\lambda}_1)$，并结合条件(H3)知，存在点 $\boldsymbol{x}_1\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\lambda}_1)$，$\boldsymbol{x}_2\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\lambda}_2)$，有

$$\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}_1\|\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{29}$$

$$\|\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})-\boldsymbol{x}_2\|\leqslant L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{30}$$

再由条件(H3)知，存在点 $\bar{\boldsymbol{x}}_1\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\lambda}_1)$，$\bar{\boldsymbol{x}}_2\in \boldsymbol{K}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\lambda}_2)$，有

$$\|\bar{\boldsymbol{x}}_1-\boldsymbol{x}_1\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\| \tag{31}$$

$$\|\bar{\boldsymbol{x}}_2-\boldsymbol{x}_2\|\leqslant L_1\|\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\| \tag{32}$$

据点 $\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$，$\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$，存在点 $\boldsymbol{y}_1\in \boldsymbol{F}(\bar{\boldsymbol{x}}_2,\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu})\cap(-(\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C}))))$，$\boldsymbol{y}_2\in \boldsymbol{F}(\bar{\boldsymbol{x}}_1,\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu})\cap(-(\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C}))))$，结合式(28)，有

$$\begin{aligned}
&\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\|\leqslant \\
&\inf_{f\in \boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu})} d(\boldsymbol{f},\boldsymbol{y}_1)+ \\
&\inf_{g\in \boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu})} d(\boldsymbol{g},\boldsymbol{y}_2)\leqslant \\
&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\bar{\boldsymbol{x}}_2,\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}))+ \\
&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\bar{\boldsymbol{x}}_1,\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}))\leqslant \\
&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}))+ \\
&\rho(\boldsymbol{F}(\boldsymbol{x}_2,\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\bar{\boldsymbol{x}}_2,\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{\mu}))+ \\
&\rho(\boldsymbol{F}(\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}))+ \\
&\rho(\boldsymbol{F}(\boldsymbol{x}_1,\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}),\boldsymbol{F}(\bar{\boldsymbol{x}}_1,\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{\mu}))
\end{aligned}$$

再注意到条件(T5)，并结合式(29)～(32)，得

$$\begin{aligned}
&\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\|\leqslant \\
&L_5\|\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})-\boldsymbol{x}_2\|+L_5\|\boldsymbol{x}_2-\bar{\boldsymbol{x}}_2\|+ \\
&L_5\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}_1\|+L_5\|\boldsymbol{x}_1-\bar{\boldsymbol{x}}_1\|\leqslant \\
&2L_5L_1\|\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\|+ \\
&2L_5L_2\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|
\end{aligned}$$

据条件(T7)，知

$$\|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\|\leqslant\frac{2L_2L_5}{1-2L_1L_5}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|=L_\lambda^D\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

又注意到点 $\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$ 任意性，有

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})}\ \inf_{\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} \|\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})-\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\|\leqslant L_\lambda^D\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

再结合 $d(\cdot,\cdot)$ 的定义，知

$$\sup_{\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})} d(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))= \sup_{\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in \boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\backslash \boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} d(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{x}(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant$$

$$L_{\lambda}^{D}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{33}$$

类似可证

$$\sup_{x(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in S_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} d(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_{\lambda}^{D}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{34}$$

据式(33)与式(34)知,式(27)成立.

情形2 若 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$ 或 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\subset\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$,不妨假设 $\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\subset\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu})$,据 $d(\cdot,\cdot)$ 的定义,得

$$\sup_{x(\boldsymbol{\lambda}_1,\boldsymbol{\mu})\in S_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu})} d(\boldsymbol{x}(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))=0 \tag{35}$$

类似情形1的论证过程知式(33)成立.结合式(33)与式(35)知,式(27)成立.

定理9 假若条件(T1)、(T3)、(H3)、(T5)、(T6)、(T7)、(H7)成立,则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1)$,$(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant L_{\mu}^{D}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|+L_{\lambda}^{D}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\| \tag{36}$$

证明 由 $\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_2))+\rho(\boldsymbol{S}_1^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_2),\boldsymbol{S}_1^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))$,并结合式(19)与式(27)知,式(36)成立.

现给出例子检验定理9的结论.

例3 设 $\boldsymbol{X}=\boldsymbol{Y}=\boldsymbol{R},\boldsymbol{M}=\boldsymbol{N}=[0,1],\boldsymbol{C}=\boldsymbol{R}_+=[0,+\infty),\boldsymbol{F}(y,x,\lambda)=[(\lambda-x-1)\lambda y/3+\lambda-x+2,\lambda y/3+6],\boldsymbol{K}(x,\lambda)=[(\lambda^2+x)/2,1]$,则 $\boldsymbol{E}(\lambda)=[(1+\lambda)^2/15,2]$. 取 $\lambda_0=\dfrac{1}{2}$,$\boldsymbol{U}(\lambda_0)=\boldsymbol{M}=[0,1]$,则对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$,有 $\boldsymbol{F}(x,y,\lambda)\neq\boldsymbol{F}(y,x,\lambda),\boldsymbol{E}(\lambda)=\boldsymbol{E}(\boldsymbol{U}(\lambda_0))=\boldsymbol{S}_1(\lambda,\lambda)=[\lambda^2,1]$,$\boldsymbol{S}_1^D(\lambda,\lambda)=[\lambda,1]$,且 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\setminus\boldsymbol{S}_1^D(\lambda,\lambda)=[\lambda^2,\lambda]$.

易知,对任何的 $\lambda\in\boldsymbol{U}(\lambda_0)$,有 $\boldsymbol{S}_1^D(\lambda,\mu)\subset\boldsymbol{E}(\lambda)$ 为非空紧子集,$\boldsymbol{K}(\cdot,\cdot)$ 在 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\times\boldsymbol{U}(\lambda_0)$ 上为 $\dfrac{1}{2}$-1-Lipschitz 一致连续的,对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x\in\boldsymbol{E}(\lambda)$,$\boldsymbol{F}(\cdot,x,\lambda)$ 在 $\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\lambda_0)),\boldsymbol{U}(\lambda_0))$ 上为 $\dfrac{1}{3}$-Lipschitz 连续的,以及对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x,y\in\boldsymbol{E}(\lambda)$,$\boldsymbol{F}(x,y,\cdot)$ 在 $\boldsymbol{U}(\lambda_0)$ 上为 $\dfrac{4}{3}$-Lipschitz 连续的,且对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $y\in[\lambda^2,\lambda]$,取点 $x_1=\dfrac{1}{2}\in\boldsymbol{S}_1^D(\lambda_0,\lambda_0)$,有

$$\inf_{f\in F(x_1,y,\mu)} d(\boldsymbol{f},-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in F(y,x_1,\mu)} d(\boldsymbol{g},-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))=$$
$$\inf_{f\in[0,8]} d(\boldsymbol{f},-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in[7(0.5-y)/6,8]} d(\boldsymbol{g},-\boldsymbol{Y}\setminus(-\mathrm{int}(\boldsymbol{C})))\geqslant$$
$$\frac{7}{6}\left\|y-\frac{1}{2}\right\|\geqslant\left\|y-\frac{1}{2}\right\|$$

此外,易见 $L_1L_5=\dfrac{1}{2}\cdot\dfrac{1}{3}=\dfrac{1}{6}\in[0,1/2)$.

类似问题(DPSVQEP1)解的 Lipschitz 连续性的分析过程,易知问题(DPSVQEP2)解的 Lipschitz连续性定理.

定理10 假若条件(T2)、(T4)、(H3)、(T5)、(T6)、(H7)成立,则对任意的点 $(\boldsymbol{\lambda},\boldsymbol{\mu}_1)$,$(\boldsymbol{\lambda},\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\rho(\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu}_1),\boldsymbol{S}_2^D(\boldsymbol{\lambda},\boldsymbol{\mu}_2))\leqslant L_{\mu}^{D}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|$$

定理11 假若条件(T2)、(T4)、(H3)、(T5)、(T7)成立,则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu})$,$(\boldsymbol{\lambda}_2,\boldsymbol{\mu})\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\rho(\boldsymbol{S}_2^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}),\boldsymbol{S}_2^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}))\leqslant L_{\lambda}^{D}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

定理12 假若条件(T2)、(T4)、(H3)、(T5)、(T6)、(T7)、(H7)成立,则对任意的点 $(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1)$,$(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2)\in\boldsymbol{U}(\boldsymbol{\lambda}_0)\times\boldsymbol{V}(\boldsymbol{\mu}_0)$,有

$$\rho(\boldsymbol{S}_2^D(\boldsymbol{\lambda}_1,\boldsymbol{\mu}_1),\boldsymbol{S}_2^D(\boldsymbol{\lambda}_2,\boldsymbol{\mu}_2))\leqslant L_{\mu}^{D}\|\boldsymbol{\mu}_1-\boldsymbol{\mu}_2\|+L_{\lambda}^{D}\|\boldsymbol{\lambda}_1-\boldsymbol{\lambda}_2\|$$

现举例分析定理12结论的有效性.

例4 设 $\boldsymbol{X}=\boldsymbol{Y}=\boldsymbol{R},\boldsymbol{M}=\boldsymbol{N}=[0,1],\boldsymbol{C}=\boldsymbol{R}_+=[0,+\infty),\boldsymbol{F}(y,x,\lambda)=(-\infty,(\lambda^3(\lambda-x)-3)\lambda y/9+\lambda^3(\lambda-x)/3],\boldsymbol{K}(x,\lambda)=[(\lambda^2+x)/2,1]$,则 $\boldsymbol{E}(\lambda)=[(1+\lambda)^2/15,2]$,取 $\lambda_0=\dfrac{1}{2}$,$\boldsymbol{U}(\lambda_0)=\boldsymbol{M}=[0,1]$,则对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$,$\boldsymbol{F}(x,y,\lambda)\neq\boldsymbol{F}(y,x,\lambda),\boldsymbol{E}(\lambda)=\boldsymbol{E}(\boldsymbol{U}(\lambda_0))=\boldsymbol{S}_2(\lambda,\lambda)=[\lambda^2,1]$,$\boldsymbol{S}_2^D(\lambda,\lambda)=[\lambda,1]$,故 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\setminus\boldsymbol{S}_2^D(\lambda,\lambda)=[\lambda^2,\lambda]$.

易知,对任何的 $\lambda\in\boldsymbol{U}(\lambda_0)$,有 $\boldsymbol{S}_2^D(\lambda,\lambda)\subset\boldsymbol{E}(\lambda)$ 为非空紧子集,$\boldsymbol{K}(\cdot,\cdot)$ 在 $\boldsymbol{E}(\boldsymbol{U}(\lambda_0))\times\boldsymbol{U}(\lambda_0)$ 上为 $\dfrac{1}{2}$-1-Lipschitz 一致连续的,对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$ 及点 $x\in\boldsymbol{E}(\lambda)$,$\boldsymbol{F}(\cdot,x,\lambda)$ 在 $\boldsymbol{K}(\boldsymbol{E}(\boldsymbol{U}(\lambda_0)),\boldsymbol{U}(\lambda_0))$ 上为 $\dfrac{1}{3}$-Lipschitz 连续的,以及对任意的点

$\lambda\in\boldsymbol{U}(\lambda_0)$及点 $x,y\in\boldsymbol{E}(\lambda)$,$\boldsymbol{F}(x,y,\cdot)$在$\boldsymbol{U}(\lambda_0)$上为$\frac{4}{3}$-Lipschitz 连续的,且对任意的点 $\lambda\in\boldsymbol{U}(\lambda_0)$及点 $y\in[\lambda^2,\lambda]$,取点 $x_2=\frac{1}{2}\in\boldsymbol{S}_2^D(\lambda_0,\lambda_0)$,有

$$\inf_{f\in F(x_1,y,\mu)} d(\boldsymbol{f},-\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in F(y,x_1,\mu)} d(\boldsymbol{g},-\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))=\inf_{f\in[0,8]} d(\boldsymbol{f},-\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))+\inf_{g\in[7(0.5-y)/6,8]} d(\boldsymbol{g},-\boldsymbol{Y}\backslash(-\mathrm{int}(\boldsymbol{C})))\geqslant\frac{7}{6}\left\|y-\frac{1}{2}\right\|\geqslant\left\|y-\frac{1}{2}\right\|$$

此外,易见 $L_1L_5=\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}\in[0,1/2)$.

4 结　语

本文在目标函数和约束函数受参数扰动下于赋范线性空间中建立了含参集值向量拟均衡问题和对偶问题解的 Lipschitz 连续性定理. 研究结果表明,两类含参集值向量拟均衡问题和对偶问题解的 Lipschitz 连续的充分性条件具有一致性数学结构,这为研究含参集值向量拟均衡问题和对偶问题(近似)解的 Lipschitz 连续性奠定了理论依据,并为获得广义含参集值向量拟均衡问题和对偶问题(近似)解的 Lipschitz 连续性和 Hölder 连续性统一框架模型提供了研究思路,同时为研究集值向量拟均衡问题解的连通性、对偶性、近似计算等提供了理论借鉴.

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Lipschitz continuity of solutions for parametric set-valued vector quasi-equilibrium problems and dual problems

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Abstract: Lipschitz continuity of solutions for a class of parametric set-valued vector quasi-equilibrium problems and dual problems is studied in normed linear spaces. The concepts for parametric set-valued vector quasi-equilibrium problems and dual problems are proposed. Under the basic assumption that the constraint function has Lipschitz uniform continuity, the Lipschitz continuity sufficient theorems of solutions to parametric set-valued vector quasi-equilibrium problems and dual problems are established by using the analytical method. The appropriate examples are given to illustrate the effectiveness of the results. With the help of theoretical results, the connectivity, duality and approximate calculation of solutions to parametric set-valued vector quasi-equilibrium problems can be further studied.

Key words: parametric set-valued vector quasi-equilibrium problems; solution; set-valued mapping; Lipschitz continuity